NRIAG Journal of Astronomy and Geophysics (2012) 1, 81–86

National Research Institute of Astronomy and Geophysics

NRIAG Journal of Astronomy and Geophysics

www.elsevier.com/locate/nrjag

The tree clustering technique and the physical reality of galaxy groups

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Received 15 October 2012; accepted 29 November 2012
Available online 4 February 2013

Abstract In this paper the tree clustering technique (the Euclidean separation distance coefficients) is suggested to test how the Hickson compact groups of galaxies (HCGs) are really physical groups. The method is applied on groups of 5 members only in Hickson's catalog.

Introduction

Groups of galaxies occupy an intermediate position in the spectrum of galaxy population between individual galaxies and binary galaxies on the one hand, and rich clusters on the other hand. According to de Vaucouleurs (1975), groups contain nearly half of the galaxies in the universe, and redshift surveys of the nearby universe indicate that most galaxies occur in small groups (e.g., Holmberg, 1950; Humason et al., 1956; de Vaucouleurs, 1965; Materne, 1979; Huchra and Geller, 1982; Geller and Huckra, 1983; Tully, 1987; Nolthenius and White, 1987).

A galaxy group is called Compact Group of Galaxies (hereafter CGGs), as generally agreed, when the mean projected separation among the member galaxies is comparable to the diameters of the galaxies themselves and the group is sufficiently isolated. Several different lists of CGGs have been mentioned in the literature following different selection criteria.

There are many catalogs of galaxy groups differ from each other by various selection criteria of members in the groups (e.g., Shakhbazian, 1957; de Vaucouleurs, 1975; Turner and Gott, 1976a,b; Rose, 1977; Karachentsev et al., 1979; Hickson, 1982; Garcia, 1995; Barton et al., 1996; Allam and Tuker, 2000; Focardi and Kelm, 2002; Iovino et al., 2003; Lee et al., 2004; Crook et al., 2007; Wang et al., 2008; McConnachie et al., 2008; Popescu and Nedelia, 2008), Most of these catalogs were defined using criteria of compactness (high surface density), isolation of possible members from field galaxies and the brightest members.

Because most criteria in the group catalogs depend mainly on redshift observations, many problems in these catalogs were found such as discordant members, and galaxies that appear to be very near to each other in space. So conditions regarding redshift or density enhancement in a cluster is still puzzling and introduces many troubles.

One of the most serious troubles is the projection effect. The uncertainties in determining the radii of galaxies (R) are big especially due to uncertainties in distance determinations

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Peer review under responsibility of National Research Institute of Astronomy and Geophysics.

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http://dx.doi.org/10.1016/j.nrjag.2012.12.002

(D). The latter depends on the measured velocities (V) and the value of Hubble constant (H_0). This can be added to the fact that one is not sure if these groups are true physical groups or just projection (Sulentic, 1983). The uncertainties in V, D and H_o cause serious problems in the radii (R) of the groups and the radii of individuals which makes a relative uncertainty in R.

This confusion inspired us to introduce the present work using some discrimination and clustering techniques to study how close the astrophysical properties of any member to the other members are?

Method and technique

Our technique depends on studying some attributes of objects which seems to form a group or cataloged as group. From the beginning we shall assume that these are not clusters. If these attributes are similar or nearly equal, according to the philosophy of the technique then it may form a cluster. In such case we would assign (for this collection of objects) the name of the cluster or compact cluster depending on how close these astrophysical attributes are.

The main core of the method depends on a matrix in which columns stands for objects while its rows are concerned with the attributes of these objects. Here we can deal with some kind of attributes in the matrix which are connected physically and related intrinsically to the objects and possibly ties each object with its companion in the group (Magnitudes, color indices, absorption coefficients, etc.) organically. One can assume that, the galaxies have been formed in a process and in a medium that still tying them and reflect its presence in the strong proximity of the individuals to each other. Also dynamical attributes can be used. The method can be used lonely to prove clustering. The technique can produce criteria to assure or to refuse clustering. The potential of this method can be extended to prove membership in open star clusters and globulars as well.

This matrix enables the determination of similarity or dissimilarity between individual galaxies that may form a group. If the attributes are close to each other, we may expect clustering. If the attributes are very close or nearly equal we can expect compact clustering in its real sense.

Cluster analysis assesses the similarity between samples by measuring the Euclidean distances coefficients between the points in the attribute space. Galaxies that are similar will lie close to one another, whereas dissimilar galaxies lie far from each other in the galaxy attribute space. The choice of the distance metric to express similarity between galaxies in a data set depends on the type of measured variables (attributes) used (magnitudes, color indices,...).

The Euclidean distance coefficient (separation) is the best choice for the distance metric, because inter-point distances between the samples can be computed directly, it measures how big the similarity or dissimilarity between the attributes of objects regardless of the number.

This is most effectively defined by the Euclidean distance coefficient given by the following Eq. (1).

$$e_{jk} = \sqrt{\sum_{i=1}^{3}(X_{ij} - X_{ik})^2} \quad (1)$$

This means that to compute e_{jk} for two objects j and k we use the data in the jth and kth columns of the original data matrix Table 1. Adding a third attribute, the Euclidian distance coefficient is given by just adding a third term, i.e. a generalization of n attribute can take the form

$$e_{jk} = \sqrt{\sum_{i=1}^{n}(X_{ij} - X_{ik})^2} \quad (2)$$

Eq. (2) gives the square root of the sum of the squares of the differences of the values of the n attributes.

The average Euclidean distance coefficient d_{jk} is defined as the average of the squares of the differences, expressed as,

$$d_{jk} = \sqrt{\sum_{i=1}^{n}\left[\frac{(X_{ij} - X_{ik})^2}{n}\right]} \quad (3)$$

In these equations X_{ij} stands for the value of the ith attribute measured on the jth object and X_{ik} is the value of the ith attribute measured for the kth object (Romesburg, 1984).

This method, the UPGMA (Unweighted Pair Group Method using Arithmetic Average) is used to reanalyze 5 members groups in HCGs catalog (Hickson, 1982, 1993). The Euclidean distance coefficient, which one of the clustering techniques, is used. For this reason the technique requires that the objects are not clusters as agreed upon by astronomers but isolated bodies. Depending on the Euclidean distance coefficient, we can decide whether we are dealing with a cluster, sub cluster, twin or a triplet, etc.

We used the same criteria in Sabry et al. (2009) which suppose that;

1. Galaxies of coefficients smaller by any value than $e_{av} - \sigma$ are given the name Twin (T). The twin property is here of a relative sense, because it depends on the attributes of the groups. No standardization has been done yet.
2. Galaxies of coefficients of the order $e_{ij} < e_{av}$ given the name pair (P).
3. Coefficients ranging between $e_{av} \leqslant e_{ij} \leqslant e_{av} + \sigma$ are given the name member (M).
4. If the coefficients are $e_{av} > e_{av} + \sigma$, it is called attribute discordant galaxy (AD). It is the galaxy that, whenever its attributes enters with attributes of the other galaxies in an assembly falsifies the Euclidean coefficients.
5. To decide the triplet character, the combined Euclidean distance coefficient (CEC) should be determined. Although triplets can be seen directly from the coefficients, we found it necessary to determine the CEC to confirm the results and isolate them quantitatively.

The following Table 1 is just an example to indicate how to deal with.

For HCG 12 from catalog of Hickson (1993), a segment of the Hickson list is shown in Table 1 where, column 1 gives the attributes (Magnitude and color indices for objects (galaxies 1–5)).

Applying Eq. (2) gives the Euclidian distance coefficient which shows the degree of similarity or dissimilarity between the two objects.

We get resemblance matrix of HCG 12 in Table 2 and the attribute space of HCG 12 is shown in Fig. 1.

Table 1 The attributes of 5 galaxies in HCG 12.

Attributes	Object				
	1	2	3	4	5
Magnitude (B)	14.82	16.29	17.30	17.21	17.89
Color index (B–V)	1.71	1.76	1.74	1.74	1.68

Table 2 Resemblance matrix coefficients.

	1	2	3	4
1				
2	1.47085			
3	2.48018	1.01020		
4	2.39019	0.92022	0.09000	
5	3.16014	1.69189	0.68264	0.77233

In Table 2 we can see that

$$e_{12} = 1.47085; e_{13} = 2.48018; e_{14} = 2.39019; e_{15} = 3.16014$$

$$e_{23} = 1.01020; e_{24} = 0.92022; e_{25} = 1.69189$$

$$e_{34} = 0.09000; e_{35} = 0.68264; e_{45} = 0.77233$$

while $e_{av} = 1.46686$ and $\sigma = 0.96019$

Following our new criteria we can say that galaxies 3 and 4 make a twin, galaxies 4 and 5 make a twin too, galaxies 2 and 3 make a binary, galaxies 2 and 4 make a binary and galaxies 3 and 5 make a binary too.

Results

For 5 members groups in Hickson catalog, our results are listed in Table 3. This table contains 10 columns, successively they are, column 1 includes the group number, column 2 includes color index of the 1st object, column 3 includes the color index of the 2nd object, column 4 the blue magnitude of the 1st object, column 5 includes the blue magnitude of the 2nd object, Columns 6–9 contain the Euclidean distance coefficients, followed by its value, the average Euclidean distance coefficient of all members and the standard deviation respectively. Column 10 includes our classification about these results. In this column, T = Twin, P = Pair, TR = Triplet and AD = Attribute discordant.

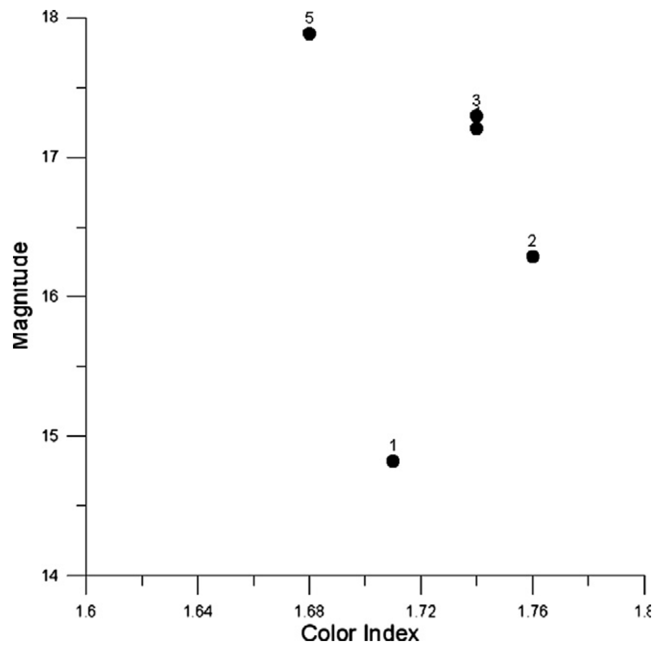

Fig. 1 Attribute space of HCG 12. Blue magnitude against color index (B–R).

Table 3 The Euclidean distance coefficients of some 5 members in Hickson compact groups' catalog.

1 HCG No.	2 $(B-R)_i$	3 $(B-R)_j$	4 B_i	5 B_j	6 and 7 Euclidian coefficients		8 e_{av}	9 σ	10 Notice
HCG 13									
	1.80	1.80	14.61	15.25	e_{12}	0.64000	1.55416	0.88421	T(1,2)
	1.80	1.71	14.61	16.24	e_{13}	1.63248			P(2,3)
	1.80	1.64	14.61	17.47	e_{14}	2.86447			P(3,4)
	1.80	1.61	14.61	17.28	e_{15}	2.67675			P(4,5)
	1.80	1.71	15.25	16.24	e_{23}	0.99408			TR(3,4,5)
	1.80	1.64	15.25	17.47	e_{24}	2.22576			
	1.80	1.61	15.25	17.28	e_{25}	2.03887			This group have two subgroups
	1.71	1.64	16.24	17.47	e_{34}	1.23199			1. Galaxies 1 and 2
	1.71	1.61	16.24	17.28	e_{35}	1.04480			2. Galaxies 3–5
	1.64	1.61	17.47	17.28	e_{45}	0.19235			
HCG 17									
	1.88	1.92	16.52	16.59	e_{12}	0.08062	1.37573	0.82108	
	1.88	1.91	16.52	17.45	e_{13}	0.93048			T(1,2)
	1.88	1.85	16.52	18.54	e_{14}	2.02022			P(1,3)
	1.88	1.90	16.52	18.98	e_{15}	2.46008			P((2,3)
	1.92	1.91	16.59	17.45	e_{23}	0.86006			$e_{15} > e_{av}$ and $e_{15} > e_{av} + \sigma$
	1.92	1.85	16.59	18.54	e_{24}	1.95126			$e_{25} > e_{av}$ and $e_{25} > e_{av} + \sigma$
	1.92	1.90	16.59	18.98	e_{25}	2.39008			$e_{35} > e_{av}$
	1.91	1.85	17.45	18.54	e_{34}	1.09165			Galaxy 5 Is Attribute
	1.91	1.90	17.45	18.98	e_{35}	1.53003			Discordant (AD)
	1.85	1.90	18.54	18.98	e_{45}	0.44283			

Table 3 (*continued*)

1 HCG No.	2 $(B-R)_i$	3 $(B-R)_j$	4 B_i	5 B_j	6 and 7 Euclidian coefficients		8 e_{av}	9 σ	10 Notice
HCG 24									
	1.69	1.54	15.23	14.89	e_{12}	0.37162	1.55013	0.92779	
	1.69	1.61	15.23	16.60	e_{13}	1.37233			T(1,2); T(4,5)
	1.69	1.61	15.23	17.46	e_{14}	2.23143			P(1,3)
	1.69	1.63	15.23	17.64	e_{15}	2.41075			P(3,4)
	1.54	1.61	14.89	16.60	e_{23}	1.71143			P(3,5)
	1.54	1.61	14.89	17.46	e_{24}	2.57095			May be we have TR(3,4,5)
	1.54	1.63	14.89	17.64	e_{25}	2.75147			And 2 subgroups
	1.61	1.61	16.60	17.46	e_{34}	0.86000			
	1.61	1.63	16.60	17.64	e_{35}	1.04019			
	1.61	1.63	17.46	17.64	e_{45}	0.18111			
HCG 37									
	1.79	1.99	12.97	14.50	e_{12}	1.54302	1.85973	0.93156	P(1,2)
	1.79	1.76	12.97	15.57	e_{13}	2.60017			P(3,4)
	1.79	0.51	12.97	15.87	e_{14}	3.16992			T(4,5)
	1.79	1.02	12.97	16.21	e_{15}	3.33024			We have 2 subgroups in this groups
	1.99	1.76	14.50	15.57	e_{23}	1.09444			Galaxies 1 and 2 in the side
	1.99	0.51	14.50	15.87	e_{24}	2.01675			And triplet galaxies 3–5
	1.99	1.02	14.50	16.21	e_{25}	1.96596			
	1.76	0.51	15.57	15.87	e_{34}	1.28550			
	1.76	1.02	15.57	16.21	e_{35}	0.97837			
	0.51	1.02	15.87	16.21	e_{45}	0.61294			
HCG 40									
	1.75	1.84	13.44	14.58	e_{12}	1.14355	1.47025	0.88666	P(1,2)
	1.75	2.00	13.44	15.15	e_{13}	1.72818			P(1,4)
	1.75	1.56	13.44	14.53	e_{14}	1.10644			T(2,4)
	1.75	1.84	13.44	16.69	e_{15}	3.25125			P(3,4)
	1.84	2.00	14.58	15.15	e_{23}	0.59203			$e_{15} > e_{av}$ and $e_{15} > e_{av} + \sigma$
	1.84	1.56	14.58	14.53	e_{24}	0.28443			$e_{25} > e_{av}$and $e_{25} < e_{av} + \sigma$
	1.84	1.84	14.58	16.69	e_{25}	2.11000			$e_{35} < e_{av}$
	2.00	1.56	15.15	14.53	e_{34}	0.76026			Galaxy 5 may be Attribute discordant
	2.00	1.84	15.15	16.69	e_{35}	1.54829			
	1.56	1.84	14.53	16.69	e_{45}	2.17807			
HCG 50									
	2.17	2.14	18.40	18.50	e_{12}	0.10440	0.68811	0.40189	T(1,2)
	2.17	2.14	18.40	19.30	e_{13}	0.90050			T(3,4)
	2.17	2.25	18.40	19.20	e_{14}	0.80399			P(3,5)
	2.17	2.13	18.40	19.70	e_{15}	1.30062			P(4,5)
	2.14	2.14	18.50	19.30	e_{23}	0.80000			TR(3,4,5)
	2.14	2.25	18.50	19.20	e_{24}	0.70859			We have 2 subgroups
	2.14	2.13	18.50	19.70	e_{25}	1.20004			1. galaxies 1 and 2
	2.14	2.25	19.30	19.20	e_{34}	0.14866			2. triplet galaxies 3–5
	2.14	2.13	19.30	19.70	e_{35}	0.40013			
	2.25	2.13	19.20	19.70	e_{45}	0.51420			
HCG 56									
	1.51	1.43	15.24	14.50	e_{12}	0.74431	1.04279	0.55636	P(1,2)
	1.51	1.52	15.24	15.37	e_{13}	0.13038			T(1,3)
	1.51	1.62	15.24	16.52	e_{14}	1.28472			P(1,5)
	1.51	1.20	15.24	16.23	e_{15}	1.03740			P(2,3)
	1.43	1.52	14.50	15.37	e_{23}	0.87464			P(3,5)
	1.43	1.62	14.50	16.52	e_{24}	2.02892			P(4,5)
	1.43	1.20	14.50	16.23	e_{25}	1.74522			We notice that
	1.52	1.62	15.37	16.52	e_{34}	1.15434			$e_{24} > e_{av}$and $e_{24} > e_{av} + \sigma$
	1.52	1.20	15.37	16.23	e_{35}	0.91761			$e_{25} > e_{av}$and $e_{25} > e_{av} + \sigma$
	1.62	1.20	16.52	16.23	e_{45}	0.51039			May be galaxy 4 is AD

Table 3 *(continued)*

1 HCG No.	2 $(B-R)_i$	3 $(B-R)_j$	4 B_i	5 B_j	6 and 7 Euclidian coefficients		8 e_{av}	9 σ	10 Notice
HCG 58									
	1.31	1.57	13.56	13.40	e_{12}	0.30529	0.81414	0.42631	T(1,2)
	1.31	1.27	13.56	13.83	e_{13}	0.27295			T(1,3)
	1.31	1.48	13.56	14.49	e_{14}	0.94541			P(2,3)
	1.31	1.20	13.56	14.86	e_{15}	1.30464			P(3,4)
	1.57	1.27	13.40	13.83	e_{23}	0.52431			TR(1,2,3)
	1.57	1.48	13.40	14.49	e_{24}	1.09371			
	1.57	1.20	13.40	14.86	e_{25}	1.50615			
	1.27	1.48	13.83	14.49	e_{34}	0.69260			
	1.27	1.20	13.83	14.86	e_{35}	1.03238			
	1.48	1.20	14.49	14.86	e_{45}	0.46400			
HCG 65									
	1.59	1.63	13.71	14.54	e_{12}	0.83096	0.66401	0.43781	P(2,3)
	1.59	1.66	13.71	14.83	e_{13}	1.12219			P(2,4)
	1.59	1.77	13.71	14.94	e_{14}	1.24310			P(2,5)
	1.59	1.43	13.71	15.05	e_{15}	1.34952			T(3,4)
	1.63	1.66	14.54	14.83	e_{23}	0.29155			P(3,5)
	1.63	1.77	14.54	14.94	e_{24}	0.42379			P(4,5)
	1.63	1.43	14.54	15.05	e_{25}	0.54781			Galaxy 1 is Attribute discordant
	1.66	1.77	14.83	14.94	e_{34}	0.15556			
	1.66	1.43	14.83	15.05	e_{35}	0.31828			
	1.77	1.43	14.94	15.05	e_{45}	0.35735			
HCG 68									
	1.63	1.63	11.84	12.24	e_{12}	0.40000	1.40014	0.80186	T(1,2)
	1.63	1.10	11.84	11.93	e_{13}	0.53759			T(1,3)
	1.63	1.30	11.84	13.73	e_{14}	1.91859			P(2,3)
	1.63	1.40	11.84	14.22	e_{15}	2.39109			T(4,5)
	1.63	1.10	12.24	11.93	e_{23}	0.61400			We have 2 subgroups
	1.63	1.30	12.24	13.73	e_{24}	1.52611			Galaxies 1, 2 and 3 in the
	1.63	1.40	12.24	14.22	e_{25}	1.99331			Subgroup and galaxies 4, 5
	1.10	1.30	11.93	13.73	e_{34}	1.81108			make new subgroups.
	1.10	1.40	11.93	14.22	e_{35}	2.30957			galaxies 4 and 5 may be out
	1.30	1.40	13.73	14.22	e_{45}	0.50010			of this groups
HCG 74									
	1.90	1.88	14.06	15.07	e_{12}	1.01020	1.74692	1.00009	P(1,2)
	1.90	1.83	14.06	16.10	e_{13}	2.04120			P(2,3)
	1.90	1.85	14.06	16.32	e_{14}	2.26055			P(2,4)
	1.90	1.79	14.06	17.80	e_{15}	3.74162			T(3,4)
	1.88	1.83	15.07	16.10	e_{23}	1.03121			P(3,5)
	1.88	1.85	15.07	16.32	e_{24}	1.25036			P(4,5)
	1.88	1.79	15.07	17.80	e_{25}	2.73148			This is very close and real
	1.83	1.85	16.10	16.32	e_{34}	0.22091			physical
	1.83	1.79	16.10	17.80	e_{35}	1.70047			groups
	1.85	1.79	16.32	17.80	e_{45}	1.48122			
HCG 83									
	1.76	1.75	15.99	16.04	e_{12}	0.05099	1.44991	0.78402	T(1,2)
	1.76	1.16	15.99	16.70	e_{13}	0.92957			P(1,3)
	1.76	1.04	15.99	17.91	e_{14}	2.05056			P(2,3)
	1.76	1.69	15.99	18.40	e_{15}	2.41102			P(3,4)
	1.75	1.16	16.04	16.70	e_{23}	0.88527			P(4,5)
	1.75	1.04	16.04	17.91	e_{24}	2.00025			Because
	1.75	1.69	16.04	18.40	e_{25}	2.36076			$e_{15} > e_{av}$ and $e_{15} > e_{av} + \sigma$
	1.16	1.04	16.70	17.91	e_{34}	1.21593			and
	1.16	1.69	16.70	18.40	e_{35}	1.78070			$e_{25} > e_{av}$ and $e_{25} > e_{av} + \sigma$
	1.04	1.69	17.91	18.40	e_{45}	0.81400			$e_{35} > e_{av}$
									Galaxy 5 may be Attribute discordant

Table 3 *(continued)*

1 HCG No.	2 $(B-R)_i$	3 $(B-R)_j$	4 B_i	5 B_j	6 and 7 Euclidian coefficients		8 e_{av}	9 σ	10 Notice
HCG 97									
	1.76	1.57	14.16	14.83	e_{12}	0.69642	0.95941	0.77188	P(1,2)
	1.76	1.51	14.16	14.54	e_{13}	0.45486			P(1,3)
	1.76	1.50	14.16	14.45	e_{14}	0.38949			P(1,4)
	1.76	1.67	14.16	16.31	e_{15}	2.15188			P(2,3)
	1.57	1.51	14.83	14.54	e_{23}	0.29614			P(2,4)
	1.57	1.50	14.83	14.45	e_{24}	0.38639			T(3,4)
	1.57	1.67	14.83	16.31	e_{25}	1.48337			Because
	1.51	1.50	14.54	14.45	e_{34}	0.09055			$e_{15} > e_{av}$and $e_{15} > e_{av} + \sigma$
	1.51	1.67	14.54	16.31	e_{35}	1.77722			and
	1.50	1.67	14.45	16.31	e_{45}	1.86775			$e_{25} > e_{av}$and $e_{25} < e_{av} + \sigma$
									$e_{35} > e_{av}$ and $e_{35} > e_{av} + \sigma$
									$e_{45} > e_{av}$ and $e_{45} > e_{av} + \sigma$
									Galaxy 5 is attribute discordant

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