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FULL LENGTH ARTICLE

Overlap technique for fitting the range residuals of the SLR data

Susan Wassem Samwel *, Yousry Shafik Hanna [1], Makram Ibrahim [1], Adel Tawfik Roman [1]

National Research Institute of Astronomy and Geophysics (NRIAG), Egypt

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Abstract The present paper concerns fitting the range residuals of the Satellite Laser Ranging (SLR) data using the overlap technique. The range residuals are characterized by its randomly distributed data points. The method used for fitting is represented. The results are compared with those obtained when fitting the whole data using Chebyshev polynomial and Least Squares Method. The later is already applied in Helwan SLR station. We have found that the overlap technique allows us to use low degree polynomials in order to fit large number of data points and, at the same time, provides better standard deviations, which may be significant, when one wants to reach best fitting. Therefore, the results obtained have approved the choice of the overlap technique in dealing with the SLR range residuals, in comparable with fitting the whole data using either Chebyshev polynomial or the Least Squares Method.

1. Introduction

The task of the present work is to deal with the problem of data fitting. Needless to say that the process of data smoothing is considered the first step for any analysis or further scientific study. There are two main approaches for data fitting: the exact fit and the best fit. The exact fit is used if the data are very accurate, such as material properties or calibration results. It is appropriate only if a small number of data points are available. The best fit is used when the accuracy of the data is not very high such as the experimental results. It is used for fitting large number of data points (Jaluria, 2007). Polynomial interpolation is the most direct method used for data fitting. It is characterized by its simplicity (Nielsen, 1964; Samwel et al., 2005). However, fitting data with high degree polynomial may

* Corresponding author. Tel.: +20 225560645; fax: +20 225548020.
E-mail addresses: samwelsw@nriag.sci.eg, samwelsw@yahoo.com (S.W. Samwel), yousry_hanna@yahoo.com (Y.S. Hanna), makikh@yahoo.com (M. Ibrahim), atroman635@gmail.com (A.T. Roman).
[1] Tel.: +20 225560645; fax: +20 225548020.
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suffer from noticeable error which up rises due to the nature of the approximation. This disadvantage becomes particularly apparent when dealing with large number of data points which is the case of the SLR range residuals. Although a larger polynomial degree should produce a more accurate solution, in practice, approximations with higher order polynomials are more sensitive to roundoff errors. This sensitivity to roundoff errors can destroy the accuracy of a solution for large degree polynomials (Gottlieb and Jung, 2009; Lindfield and Penny, 2012). Angeles (2014), stated that an increase in the number of data points is met by a high degree polynomial. Finding the coefficients of the interpolating polynomial requires solving a system of linear equations which is corrupted with a relative roundoff error that is roughly equal to the relative roundoff data error multiplied by an amplification factor that is known as the condition number of the system matrix. As we increase the polynomial degree, the associated condition number rapidly increases. So, in order to cope with this problem, the overlap technique can be used as an alternative to higher-degree polynomial.

The present work concerns fitting the SLR range residuals i.e. the Observed–Computed (O–C) residuals. The SLR range residuals express the difference between the observed and computed satellite ranges, which is often expressed as the O–C range residuals (Ibrahim et al., 2004). It is characterized by its random distribution with the very small variation between the maximum and minimum values (Samwel et al., 2004). The SLR range residuals are used as parameter for Precise Orbit Determination (POD) of satellites (Botai, 2013).

In the subsequent sections, the SLR range residuals are fitted using both Chebyshev polynomial for fitting the whole data, and the overlap technique. The later technique, the overlap technique, is based on dividing the whole data points into subintervals of equal times, where the end time of the first subinterval is the start time of the second subinterval. The Chebyshev polynomial is also used for fitting the data of the subintervals. Section two represents briefly the Chebyshev polynomial, and the overlap technique. In section three, we use the Chebyshev polynomial for fitting the SLR range residuals of satellites AJISAI, TOPEX, and BEACON-C in two different methods. The first method is by applying the Chebyshev polynomial for fitting the whole data points. The second method is by applying the Chebyshev polynomial for fitting the data of each subinterval of the SLR range residuals using the overlap technique. Section four represents a comparison of the results obtained from the two methods of fitting, with those obtained using the Least Squares Method which is already used in Helwan-SLR station. Finally, the main results obtained are summarized as a conclusion in section five.

2. Data fitting techniques

2.1. Approximation with Chebyshev polynomials

One of the interpolating polynomials is the Chebyshev polynomial, which has been investigated by Chebyshev in the 19th century. Its importance appears in its rapid convergence, which has a great deal both in the reduction of computational time and in the estimation of the accurate upper bound error (cf. Fox and Parker, 1968).

Chebyshev polynomial is a polynomial of degree n in $\cos\theta$. Its values are limited in the closed interval $[-1,1]$ with leading coefficient one. It takes the form

$$f(x) = \cos\theta, \quad \cos\theta = x, \quad -1 \leqslant x \leqslant 1 \tag{1}$$

2.2. Overlap technique and its characteristics

The overlap technique is based on dividing the whole data points into subintervals of equal times where the end time of the first subinterval is the start time of the second subinterval. The Chebyshev polynomial is used for fitting the data of the subintervals.

Let at the interval y_i, there are n_i measurements x_{ij}; ($i = 1, 2, \ldots, m$; $j = 1, 2, \ldots, n_i$) of the independent variable x which represents the differences between the observed and the computed SLR data. Then, for arithmetic mean of the united interval y_i and y_{i+1} we have the following:

$$\bar{x}_i = \bar{x}(y_i, y_{i+1}) = \frac{1}{a_i}\left(\sum_{j=1}^{n_i} x_{ij} + \sum_{j=1}^{n_{i+1}} x_{i+1,j}\right) \tag{2}$$

and for the unbiased estimator we have

$$\widehat{S}_i^2 = D\bar{x}(y_i, y_{i+1}) = \frac{1}{a_i - 1}\left[\sum_{j=1}^{n_i}(x_{ij} - \bar{x}_i)^2 + \sum_{j=1}^{n_{i+1}}(x_{i+1,j} - \bar{x}_i)^2\right] \tag{3}$$

where D is a symbol used for unbiased estimator of the dispersion, $a_i = n_i + n_{i+1}$ and for $i = m$, we take $i + 1 = 1$.

The first and second summations on the right hand side of Eq. (3) are the variations within united intervals as they involve the squares of the deviations of x_{ij} and $x_{i+1,j}$ from the intermediate normal point $\bar{x}_i$ given by Eq. (2)

In order to fit the data of the subintervals using Chebyshev polynomial, let y_i, $i = 1, 2, \ldots, n$ represent observational measurements taken at times t_i, $i = 1, 2, \ldots n$.

Suppose that these data are divided into subintervals of equal times. Two consecutive groups, $y_{ij}, i = 1, 2$ and $j = 0, 1, \ldots, n_i$ are selected. The closed time intervals $[t_{i0}, t_{in}]$ are mapped into the closed intervals $[-1, 1]_i$, using the following transformation (cf. Henrici, 1963)

$$x_{ij} = \frac{2t_{ij} - t_{i0} - t_{in}}{t_{in} - t_{i0}} \quad j = 0, 1, 2, \ldots n \tag{4}$$

Now two independent Chebyshev polynomials, as represented in (Eq. (1)) are constructed for y_{ij} as functions of variable x as defined by (Eq. (4)) are given in the following form

$$p_i(x) = \sum_{k=0}^{m_i} \tilde{a}_{ik} f_{ik}(x) \tag{5}$$

where $f_{ik}(x)$ are the Chebyshev polynomials (cf. Fox and Parker, 1968).

In order to join the two constructed interpolating polynomials for the two successive intervals, the coefficients a_{2k}, ($k = 0, 1, \ldots, m_2$) have to be determined. Analysis of the variations within and between the overlapping intervals is found and discussed by Hanna (2002), Samwel et al. (2005) and Hanna et al. (2012).

Table 1 Part of the data of satellite Ajisai.

Time of observations			Range residuals (μs)
HH	MM	Sec.	
21	32	59.20029	0.05
21	32	59.60029	0.051
21	33	2.400294	0.052
21	33	12.00029	0.056
21	33	16.80029	0.059
21	33	22.80029	0.066
21	33	23.80029	0.062
21	33	25.0003	0.062
21	33	30.20029	0.064
21	33	31.4003	0.065
21	33	32.40029	0.066

3. Analysis of the SLR range residuals

The procedure of fitting data is achieved by carrying out a program that is constructed in order to reach the required accuracy at which we call a fit is good. This program is written in FORTRAN language. The program starts with a polynomial of 1st degree ($L = 1$) and computes the coefficients of the Chebyshev polynomial and then is followed by the generated polynomial. Hence, the deviation of the data fitting and its standard deviation are calculated. Then testing in sequence the difference between the current values of standard deviation assigned to the recurrent polynomials up till the difference becomes smaller than the proposed value (5×10^{-7}). The achieved polynomial degree is expected to be the best one and hence the program stops.

In the present section, the Chebyshev polynomial is used for fitting the whole SLR range residuals of three observed satellites, namely AJISAI (AJ), TOPEX (Tp), and BEACON-C (BC). In addition, the overlap technique is applied to the same data. Table 1, represents part of the data of satellite AJISAI which is used in the present study. The first column represents the time of observations in Hour (HH), Minutes (MM), and Seconds (Sec.). The second column represents the SLR range residuals (O–C) in microsecond (μs).

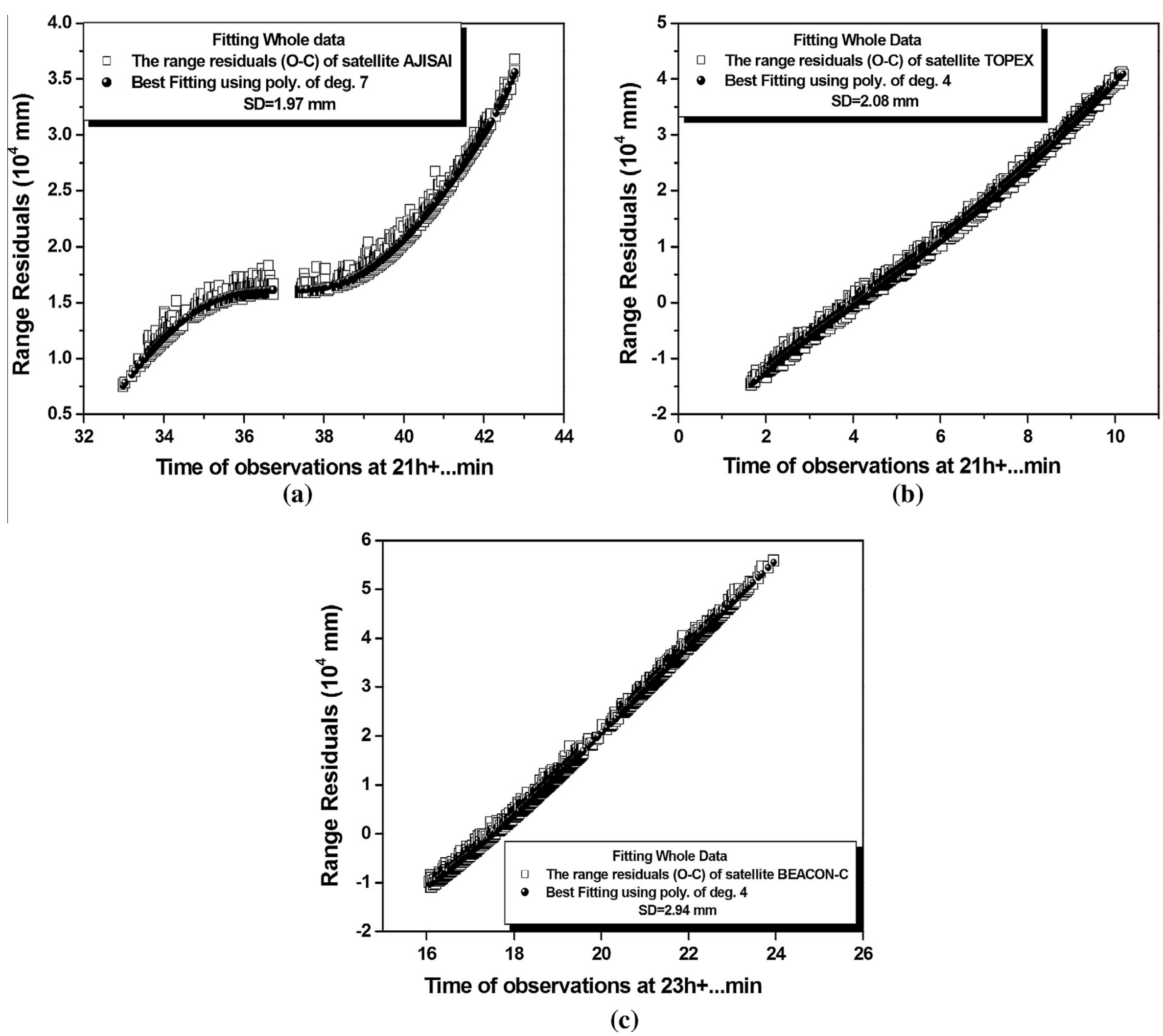

Figure 1 The range residuals with the best fitting achieved using Chebyshev polynomial for satellites (a) AJISAI, (b) TOPEX, and (c) BEACON-C.

3.1. Fitting the whole SLR range residuals using Chebyshev polynomial

Using Chebyshev polynomial, the SLR range residuals of satellites AJISAI, TOPEX, and BEACON-C, which has been observed by Helwan SLR station (Ibrahim, 2011), are fitted. The satellite AJISAI (8606101) has been observed on 2-9-2000. The data taken in case of satellite AJISAI in this observation consist of 930 data points. The whole data of the satellite AJISAI are fitted using a polynomial of degree 7. The standard deviation (SD) of the data fitting is computed to be nearly 1.97 mm. The satellite Topex (9205201) has been observed on 15-4-2000. The data taken for the satellite Topex consist of 1482 data points. The whole data of the satellite TOPEX are fitted using a polynomial of degree 4. The corresponding standard deviation (SD) is computed to be nearly 2.08 mm. Satellite Beacon-C (6503201) has been observed on 4-9-2000. The whole data taken for the satellite Beacon-C consist of 915 data points. The whole data are fitted using a polynomial of degree 4 with corresponding standard deviation of 2.94 mm. Fig. 1 represents the whole range residuals of satellites AJISAI, Topex and BEACON-C with their best fitting achieved using Chebyshev polynomial. The x-axis represents the time of observation and the y-axis represents the SLR range residuals in mm.

3.2. Fitting the SLR range residuals using overlap technique

In the present subsection, the range residuals of satellites AJISAI, TOPEX and BEACON-C mentioned in Section 3.1, are fitted using the overlap technique. For the purpose of overlap technique, the whole data of satellites AJISAI, TOPEX, and BEACON-C are divided, for instance, into six subintervals. For each satellite, the subintervals are of equal time ranges. It is found that by applying the overlap technique the range residuals of satellite AJISAI are fitted with an average polynomial degree equal 3 with a corresponding standard deviation of about 0.97 mm. Also the range residuals of satellites TOPEX and BEACON-C are fitted using polynomials of degrees 3 and 2 with corresponding standard deviations of nearly 1.29 mm and 1.99 mm respectively. Fig. 2 represents one of the subintervals for the satellites AJISAI,

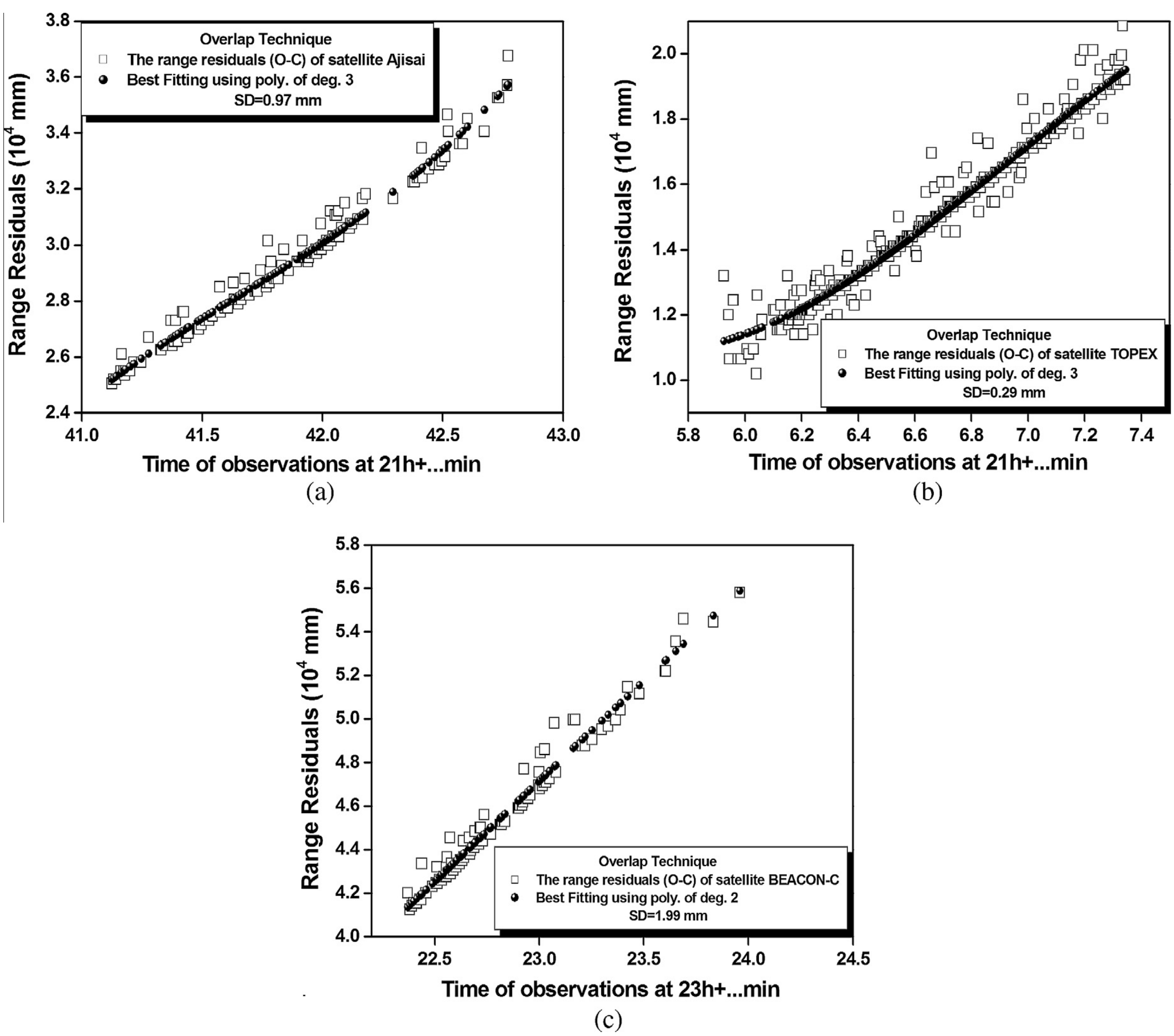

Figure 2 One of the subinterval of the range residuals and their best fitting using overlap technique for satellites (a) AJISAI, (b) TOPEX, and (c) BEACON-C.

Table 2 The polynomial degree and the corresponding standard deviation for the range residuals of the satellites AJISAI, TOPEX and BEACON-C obtained by fitting the whole data using the Least Squares Method and the Chebyshev polynomial and by applying the overlap technique.

Satellite name	Fitting the whole data				Overlap technique	
	Least Squares Method (Helwan SLR station)		Chebyshev poly.			
	Polynomial degree	Standard deviation (mm)	Polynomial degree	Standard deviation (mm)	Polynomial degree	Standard deviation (mm)
AJISAI	7	20.8	7	1.97	2	1.93
TOPEX	4	23.4	4	2.08	2	1.95
BEACON-C	7	15.4	4	2.94	2	2.86

TOPEX, and BEACON-C and their best fitting using overlap technique.

4. Comparison of the obtained results of the two methods with the Least squares method

In the present section, a comparison of the results obtained by fitting the whole data of the SLR range residuals using the Least Squares Method (Samwel et al., 2005) and the Chebyshev Polynomial, with those obtained by dividing the data into a number of subintervals using the overlap technique for the three satellites AJISAI, TOPEX and Beacon-C is represented in Table 2.

From Table 2, we can see that using the overlap technique for fitting the range residuals of the satellites AJISAI, TOPEX, and BEACON-C, allows us to use lower polynomial degree, incomparable with the other two methods, and still provides better standard deviation.

5. Discussion and conclusion

In the present study, we concern fitting the SLR range residuals using the overlap technique. The results are compared to those obtained by fitting the whole data using the Chebyshev polynomial and the Least Squares method. We found that:

– Fitting the SLR range residuals using Chebyshev polynomial provides better results than those obtained by applying the Least Squares Method used in Helwan SLR station.
– Fitting the SLR range residuals using the overlap technique provides better results than those obtained by applying the Chebyshev polynomial for fitting the whole data or by applying the Least Squares Method. The overlap technique allows us to use low degree polynomials in order to fit large number of data points and provides better standard deviations which may be significant when one, wants to reach best fitting.
– Hanna et al. (2012), applied the overlap technique on SLR range which is characterized by its smoothly varying behavior, i.e. it is characterized by its roly-poly behavior and approved that the overlap technique is a good choice for fitting this kind of data.

Hence, we conclude that the overlap technique is considered a best choice for fitting data both with roly-poly behavior (SLR range) and with randomly distribution behavior (SLR range residuals), in comparable with the other two methods mentioned above.

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