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Naked eye determination of the dawn for Sinai and Assiut of Egypt



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Abstract Twilight observations were carried out by naked eye in the period (2010–2012) for north Sinai (Lat. 31°4'N, Long. 32°52'E) where the background is desert, and for Assiut (Lat. 27°10'N, Long. 31°10'E) in the period (2012–2014) where the background is agricultural land. The purpose of these observations is to calculate the depression of the sun below the horizon at which the normal eye can discriminate the dawn (morning white thread) for two sites. The results indicated that this discrimination takes place at vertical sun depression angles, $D_o = 14.61^\circ$ and 13.665° at Sinai and Assiut respectively.

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1. Introduction

The human eye is a device which can receive focus and sort out the incident light, and then convert it into chemical, thermal and electrical energy which are necessary to trigger nerve propagation. About 96% of the light falling on the cornea passes on through the lens and about 4% is reflected. The eye behaves as an instrument, as though it had an arbitrary variable gain control capable of adjusting itself automatically to a condition under which the output signal is of comfortable strength. Its remarkable ability to adopt itself to the very wide range of

brightness from the day to the night sky levels makes the eye a convenient device for ordinary needs of light, color and form perceptions. The focal length of the human eye is about 20 mm, and the maximum aperture under dark adaptation is about 8 mm. The fovea centralism, which is the region of highest acuity, covers a field of 0.7° and the resolving power is about one minute of arc. At high brightness levels, where fovea vision comes into use, the threshold is determined by the minimum recognizable contrast in surface brightness. At the boundary between extended areas, this contrast is about 2%. The time constant of the eye is a function of the brightness level. It is 0.1 s for very low brightness and 25 ms at higher brightness of 0.01 lumen/cm^2 (Rosenberg, 1966 and Allen, 1973).

It has been determined experimentally that for a point source of light to be detectable, the minimum energy rate for light striking the eye must be 10^{-16} W . If we consider light of wavelength, say 500 nm, the number of photons arriving at the eye each second is 250 photons. The eye's response to

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Nomenclature

φ	the latitude of the place	n	the number of the days in the year (1 to 365)
δ	the declination of the sun	m	number of month (1 to 12)
D_o	sun vertical depression angle	d	day number in the month
H	the hour through which the earth has rotated since solar noon by degree.	$k = 2$	simple years, $k = 1$ Leap year
$E.T.$	the equation of time = mean sun time – real sun time	A_r	azimuth angle of the sun at sunrise
$\Delta\lambda$	the difference of longitude (true longitude – $Z.T.$)	M_m	measured morning time (dawn measured)
$Z.T.$	zonal time	M_c	calculated morning time (dawn calculated)
Z	zenith distance	N.E.	naked eye observation
		P.E.	photoelectric observations

intensity is logarithmic. This means that the eye's response to brightness is equal to a constant multiplied by the natural logarithm of the actual change in intensity. The logarithmic response of the eye may be demonstrated by experiments in which the intensity of an observed light source is varied, in relation to a background light of fixed intensity (Pirenne, 1967). The optical phenomena of twilight take place near the time of sunset and sunrise. It occupies the time interval separating the illumination conditions of daytime from night. The appearance of the sky under both twilight and daytime conditions is wholly governed by the optical structure of the atmosphere, particularly its interaction with sunlight. As the sun sinks toward the horizon, an increase in the optical path of its rays through the atmosphere is associated with a decrease in its brightness. This leads to a weakening in the illumination of the earth's surface by both direct and scattered light in the atmosphere. The combined luminance of the daytime conditions shows a slight dependence on the sun's altitudes. A progressive drop in the luminance begins to accelerate sharply when the sun's altitude is 5–10°, and the twilight is considered to have set in. Sky twilight observations and measurements entail considerable troubles (Roach and Gordan, 1973 and Donald Mc Gillivray, 1987).

Typical unaided eyes were a point of research by many investigators, to determine the minimum threshold that dark adapted eyes can characterize, for instance Knoll et al. (1946) and Richard (1946).

The intensity of the zenith skylight was measured for solar zenith angle of 90–106° for wavelengths 0.75 μm , 0.59 μm , 0.52 μm and 0.44 μm . The intensity of the zenith skylight at twilight changes by a factor of approximately 10^6 in the blue and 10^5 in the red (Ashburn, 1952).

There are six general contributors to the night sky brightness: (1) integrated light from distance galaxies; (2) integrated starlight from within our galaxy; (3) zodiacal light; (4) night airglow; (5) aurora; and (6) twilight emission lines. Night airglow, aurora, and twilight emission lines are results of planets with an atmosphere and magnetic field. Zodiacal light is a result of being within a solar system. The remaining two contributors would be present anywhere within our galaxy. Night airglow is the fluorescence of the atoms and molecules in the air from photochemical excitation. It occurs primarily in a layer about 100 km above the earth and is variable, depending on sky conditions, local time, latitude, season, and solar activity. There is a component that is present at most wavelengths, called the continuum, primarily caused by nitrous oxide and

other molecules, but the major component is caused by distinct emission lines. Both components are always present, tend to increase in brightness near the horizon, and are not strongly affected by geomagnetic activity (Henden and Kaitchuck, 1982 and Benjamin Crowell, 2008).

The currently adopted rules of beginning and end of twilight in some Arabic, Islamic countries and some areas populated by some Muslims like in the U.S.A. and Europe are different. Some areas such as Pakistan and surrounding areas such as Bangladesh, Afghanistan, India and some parts of Europe fixed both twilights (Beginning and End of twilight) at 18° depression of the sun below the horizon. This value corresponds to the astronomical twilight. It should be mentioned that, when the sun depression is 18° below the horizon, the eye receives the least possible non-perceptible light in all wavelengths from the twilight. This does not enable the normal eye to distinguish any horizon. So, people in the sea depend totally on stars of the sky to find their directions. No religious signs for Beginning and End of twilight are considered. North America, Canada, parts of U.S.A. and U.K. took a value of 15°. Um Al-Qura calendar is adopting a value between 18.5° and 19° for Beginning and 22.5° for the end of twilight nowadays for the sun depression below the horizon except in Ramadan in which the value is increased to be 30°. All of the Gulf countries follow Saudi Arabia in this regard. It should be mentioned also that Um Al-Qura calendar does not follow the religious signs (e.g. the twilight thread for beginning or the minimum red light for end of twilight). Egypt, some African countries, Syria, Iraq and Lebanon follow the published values of the Egyptian General Authority of Survey 19.5° for beginning and 17.5° for end of twilight (Hassan et al., 2009).

The authors published many articles in this field, which reported that the beginning of twilight according to the photoelectric measurements at different sites in Egypt is in the range $14 \leq D_o \leq 15^\circ$ (Issa and Hassan (2008a,b,c) in Bahria, Issa et al. (2011) in Kottamia, Hassan et al. (2013) in Matrouh and Hassan et al. (2014a,b) in Bahria). By naked eye observations on the four locations in Egypt (Matrouh, Bahria, Kottamia and Aswan), the mean vertical sun depression for observing the dawn was found to be $D_o = 14.7^\circ$ (Hassan et al. (2014a,b)).

Al Mostafa et al. (2005) studied the true dawn in the deep desert 170 km far from Riyadh city in Saudi Arabia (25°45'41"N, 74°12'10"E and 540 m height over the sea level) by two methods: naked eye observations and camera (of Nikon type) measurements. Both methods were carried out

Table 1 Summarization of the published work of observing twilight using naked eye (N.E.) and photoelectric (P.E.) instruments.

Location	Lat. N	Long. E	Elev. (m)	N.L.	Method	D_o	Authors
Riyadh (Saudi Arabia)	25°46'	74°12.16'	540	Desert	N.E. and camera	$14.6^\circ \pm 0.3$	Al Mostafa et al. (2005)
Bahria, (Egypt)	28°42.9'	29°59.82'	150	Desert	P.E.	$15.5^\circ \pm 0.5$	Issa and Hassan (2008a,b,c)
Bahria, (Egypt)	28°42.9'	29°59.82'	150	Desert	P.E.	$14 \leq D_o \leq 15.5^\circ$	Issa and Hassan (2008a,b,c)
Matrouh (Egypt)	31°0.2'	27°51'	75	Sea–Desert	P.E.	14.5°	Issa et al. (2009)
Tubruk (Libya)	32°05'	23°59'	10	Sea	N.E.	13.43°	Hassan et al. (2009)
Kottamia (Egypt)	29°55.9'	31°49.5'	470	Desert	P.E.	14.5°	Issa and Hassan (2010)
Matrouh (Egypt)	31°0.2'	27°51'	75	Sea–Desert	P.E.	$14 \leq D_o \leq 16^\circ$	Hassan et al. (2013)
Bahria, (Egypt)	28°42.9'	29°59.82'	150	Desert	P.E.	$14 \leq D_o \leq 15.5^\circ$	Hassan et al. (2014a,b)
Bahria	28°42.9'	29°59.82'	150	Desert	N.E.	$12.6 \leq D_o \leq 15^\circ$	Hassan et al. (2014a,b)
Matrouh	31°0.2'	27°51'	75	Sea–Desert	N.E.	$12.3 \leq D_o \leq 14.5^\circ$	Hassan et al. (2014a,b)
Kottamia	29°55.9'	31°49.5'	470	Desert	N.E.	$14.46 \leq D_o \leq 14.86^\circ$	Hassan et al. (2014a,b)
Aswan	23°48.22'	32°29.5'	250	Desert	N.E.	$12.46 \leq D_o \leq 13.96^\circ$	Hassan et al. (2014a,b)

parallel to each other. The observations were recorded during one year twice in every month by four groups consisting of two observers. The results indicated that the dawn can be resolved at sun vertical depression of $D_o = 14.6 \pm 0.3^\circ$, where the minimum depression value of resolving the dawn is at 14° and the maximum value is at 15.1° . The applied dawn now (at 19°) is considered to be the pseudo-dawn (zodiacal light).

Table 1 summarizes the results of the published work in the twilight in Saudi Arabia, Egypt and Libya.

The aim of the present work is determining the beginning of twilight (dawn) in Assiut city and Sinai Peninsula desert in Egypt by the naked eye observations.

2. Site observations and data sources

The observation of the morning twilight in the two sites, Sinai (Beer Al-Abd) and Assiut is recorded by the naked eye (N.E.) in cloudless morning twilight sky.

In Sinai, the observer was one of the authors (45 years) and the site of the observation was open and free of any light or air pollutants. The observations of Sinai used in this work were taken by this author. Six observations of them are already mentioned in his book (Mousa (2011)).

In Assiut, the observations were taken in Assiut city from a building of 27 m height above the surface of the earth which is 200 m far from the eastern coast of the Nile River. The background of the observations was an open agricultural land. The

observation process was taken by the author Hassan A.H. and with the help of his 17-years-old nephew.

Table 2 summarizes the geographical conditions of the two places, Sinai (Beer Al-Abd) and Assiut.

3. Method

Here, we give a brief description of the procedure. We recorded the local time M_m (time of the dawn) corresponding to what we believe to be the first light signal (white light thread) of early twilight. The sun's vertical depression angle (D_o) and the calculated morning twilight M_c can be calculated from the formulas (Aden and Marjorie, (1991), Peter, (1989) and Hassan et al. (2009 and 2014a,b), see Appendix A). We need to understand the concept of the duration of twilight Δt , see Appendix B.

4. The problem

The adopted value of the sun vertical depression for the dawn nowadays in Egypt (through the Egyptian General Authority of Survey) is 19.5° . Unfortunately, there is neither written document nor scientific theory support adopting this value. In spite of this, a lot of people do not differentiate between the pseudo dawn (zodiacal light) and the true dawn. The problem now is that we are trying to solve this problem through a series of researches started from 2008 until now as results of a large scale project started in 1984. Most of the Islamic countries are considering the Egyptian General Authority of Survey adoption. Other countries adopt the value of 18° . This is because these countries have no observations or research to determine the dawn time. The Islamic Society of North America (ISNA) adopts the value 15° below the horizon. This society is the closest society to the real value representing the true dawn since Patat et al. (2006) proved that the night sky level lies between 15° and 16° . Since the ordinary eye does not feel less than 250 photons (Donald (1987)), we believe that the sun vertical

Table 2 The geographical data of Sinai (Beer Al Abd) and Assiut sites.

	Latitude (ϕ)	Longitude (λ)	Elev. (m)	N.L.
Beer Al-Abd	31°04'N	32°52'E	10	Mediterranean Sea and desert
Assiut	27°10'N	31°10'E	74	Agricultural land

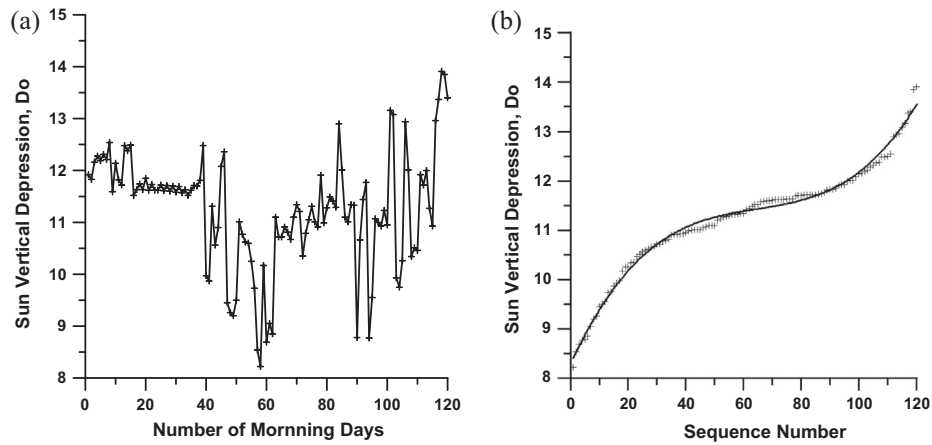


Figure 1 (a) Scatter of D_o (b), third order fitting of D_o . Variation of sun vertical depression D_o over the selected observation days at Assiut.

depression below the horizon has to be less than 15° for the dawn.

5. Results and analysis

Fig. 1a shows the variation of the naked eye sun vertical depression angle observations for 120 cloudless days at Assiut site during the two years 2013 and 2014, while Fig. 1b is the 3rd order fitting of the sorted sun vertical depression angle observations, indicating that the measured angles are homogeneously distributed around the range $11\text{--}12^\circ$.

Fig. 2 represents the distribution of the 12 bars of 0.5° width as the frequency of 120 morning twilight observations. The high frequency (statistical mode) is 27.5% at D_o interval between 11.5° and 12° in which both mean (11.247°) and median (11.375°) values are close to each other and are included in the range of nautical twilight, while the maximum of the normal distribution (the peak) is at range between 11° and 11.5° , this leads to consider the median rather than the mean.

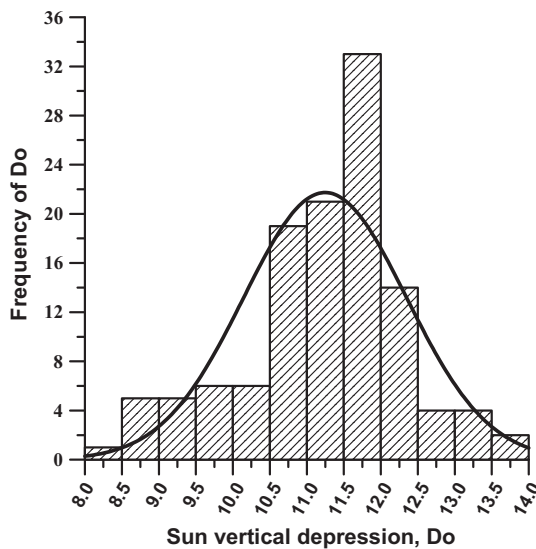


Figure 2 Frequency of D_o at Assiut.

Table 3 Date, measured morning twilight (M_m) and sun vertical depression (D_o) at Sinai.

Date	Time of M_m (h:mm)	D_o of M_m (Degree)
1/9/2010	4:15	15.18
3/9/2010	4:16	15.27
6/9/2010	4:20	14.89
7/9/2010	4:20	15.03
5/11/2010	5:08	13.02
6/11/2010	5:04	14.01
23/3/2011	4:51	13.22
24/3/2011	4:50	13.15
25/3/2011	4:48	13.30
26/3/2011	4:47	13.24

Table 4 Results of the naked eye observations of the measured morning twilight (M_m) for the sun vertical depression, D_o (degrees) at Beer Al Abd (Sinai) and Assiut.

Results	Assiut (M_m)	Sinai (M_m)
Number of values	120	10
Minimum	8.22	13.02
Maximum	13.91	15.27
Range	5.69	2.25
Mean	11.247	14.031
Median	11.375	13.655
Variance	1.223	0.912
Standard deviation, σ	1.1058	0.955
Dawn, D_o	13.655	14.61

Table 3 represents the high-visibility selected naked eye observations of 10 cloudless days as totally clear days at Sinai (Beer Al-Abd) for measured morning twilight (M_m) during the two years 2010 and 2011, while Table 4 represents the statistical comparison of the naked eye (N.E.) observations of M_m for the sun vertical depression D_o at Beer Al-Abd (Sinai) and Assiut.

Fig. 3 represents the values of maximum, minimum and median of sun vertical depression for M_m in Assiut and Sinai.

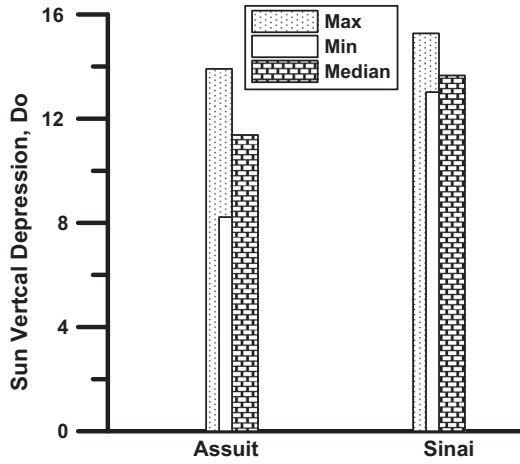


Figure 3 Maximum, Minimum and Median values of M_c as a comparison between Assiut and Sinai.

We can compare the three values for both sites from this diagram.

From Table 4, we notice some remarks for M_m in both sites. At Assiut, the dawn declares itself at sun vertical depression angle $D_o = 13.665^\circ$ (median + 2σ), where the maximum value is $D_o = 13.91^\circ$, while at Sinai it declares itself at sun vertical depression angle $D_o = 14.61^\circ$ (median + σ), where the maximum value is $D_o = 15.27^\circ$. The range value and standard deviation have lower value in Sinai (2.25°) than in Assiut (5.69°) due to the higher visibility in this site, while in Assiut, the low visibility is caused by the clammy atmosphere and transpiration.

6. Discussion

The results of Sinai indicate that the dawn (first light) occurs according to an average of the normal eye observations at desert background according to a sun depression angle $D_o = 14.61^\circ$ ($D_o = \text{median} + \sigma$) with a standard deviation of 0.955° . The maximum value of the depression D_o is 15.27° , while the minimum value is 13.02° .

The result of Sinai agrees with the results from the naked eye observations ($D_o = 14.7^\circ$) for the four locations in Egypt (Matrouh, Bahria, Kottamia and Aswan) which have the same climatological and environmental conditions in Sinai (Hassan et al., 2014a,b). These all results also agree with the observations by the photoelectric technique which indicate that the dawn declares itself according to sun depression angle values inside the interval $14 \leq D_o \leq 15^\circ$ (Issa and Hassan (2008a,b, c) at Baharia, II and III), (Issa et al. (2011) at Kottamia), (Hassan et al. (2013) at Matrouh) and (Hassan et al. (2014a, b)) at Baharia. Similarly, these results also agree with Al Mostafa et al. (2005) in Saudi Arabia which indicates that the dawn shows itself at sun vertical depression angle of $D_o = 14.6 \pm 0.3^\circ$.

The results of Sinai agree with Patat et al., (2006) in dome C (Kenyon et al. (2006)) which reported that the night sky brightness level is reached at zenith angle z between 105° and 106° . According to this study, we believe that the beginning of twilight (dawn) occurs at $Z \leq 105^\circ$ (at sun vertical depres-

sion angle $D_o \leq 15^\circ$) and the end of twilight is at $Z \geq 106^\circ$ (at sun vertical depression angle $D_o \geq 16^\circ$). The thickness of the atmospheric layers in the evening twilight is higher than the thickness of the atmospheric layers in the morning twilight because the temperature in the evening is normally 5°C higher than the temperature in the morning twilight especially in the latitudes lower than 45° . Also, spectral resolution and sky patch differ from spectra to spectra (Patat et al. (2006)). The spectra have been vertically shifted by the following amounts: $+0.5$ ($D_o = 9.4$), $+0.35$ ($D_o = 11.9$), -0.10 ($D_o = 14.8$), -1.0 ($D_o \geq 18$). With the exception of the first spectrum, which was obtained with a very low resolution ($\sim 130 \text{ \AA}$ FWHM), the remaining data allow one to detect quite a number of details. For $D_o < 12^\circ$, i.e. during the nautical twilight, the spectrum is rather different from that of the Sun, even though it shows clear solar feature, like the Call H&K lines and the G-band at about $4300 \text{ \AA}$. The Rayleigh-scattered Sun flux clearly contributes in the region bluewards of $5000 \text{ \AA}$ down to $D_o = 15^\circ$, after which the pseudo-continuum of the night sky emission takes over.

The results of Assiut indicate that, the dawn (first light) occurs according to an average of the normal eye observations at agricultural land according to a sun vertical depression angle $D_o = 13.665^\circ$ ($D_o = \text{median} + 2\sigma$) with a standard deviation of 1.06° . The maximum value of the depression D_o is 13.91° , while the minimum value is 8.22° . The range between the minimum and maximum is relatively large (5.69°), as the variation of climatic environments between the different seasons is relatively high.

These results agree with the results from the naked eye observations taken in Tubruq of Libya (Hassan et al., 2009) which indicate that the dawn shows itself at sun depression angle $D_o = 13.43^\circ$. It is important to mention here that the climatic and environmental condition in Tubruq (S_1) is a coastal area (Mediterranean), while the condition in Assiut is an agricultural land. It is already known that the high water vapor contents in the coastal surface and agricultural land surface cause low visibility in both sites.

The difference in D_o between Sinai and Assiut is 1° , which is due to the dry and clean air and very high visibility in Sinai, while in Assiut, the results show a low visibility and transparency which is due to the water vapor resulting from transpiration from agricultural background.

The compatibility between the naked eye observations and almost complete photometric observations (especially in the desert areas) opens the way for us to expand extensive observations of that kind of observations in the future.

It is clear from the researches from this work and that published from 2008 until now that the difference between the sun vertical depression value of the true dawn (14.5°) and the adopted value nowadays in Egypt (19.5°) is 5° . Converting 5° into time gives time difference of 20 min at the equator and in the Equinox and exceeds as we move away from the equator. The difference between these two values of depression has its minimum values in equinoxes and has its maximum values in the summer solstice, while its average values occur in the winter solstice (see Appendix B).

7. Conclusion

The results suggest that:

1. For Sinai, the dawn declares itself when the sun is below the horizon by $D_o = 14.61^\circ$.
2. For Assuit, the dawn declares itself when the sun is below the horizon by $D_o = 13.66^\circ$.
3. The difference between the high visibility in Sinai and the low visibility in Assuit for the sun vertical depression angle is about 1° .
4. There is a difference about 5° between the adopted sun vertical depression nowadays in Egypt 19.5° and that representing the true dawn (14.7°), which gives a time difference of 20 min at the equinoxes and exceeds with the latitude.

8. Target in the future

We are going to shed the light on the beginning and end of a false dawn (zodiacal light) as many do not distinguish between the false dawn and the true dawn.

More research about the false dawn (zodiacal light) and the true dawn, whether in kind or photometric meteorological devices, to cover new areas and in different display different climatic conditions lines. Note that with regard to dawn sincere proved in the past that our research observation-kind fully consistent between naked eye and photometric observations in kind, especially (Hassan et al., 2014a,b).

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Appendix A

To calculate the sun vertical depression angle (D_o) for the observer, we apply the following:

To determine the azimuth angle (A_r) of sunrise that we expect the emergence of dawn at them,

$$A_r = \cos^{-1} \left[\frac{\sin \delta}{\cos \varphi} \right]$$

To convert the time recorded of dawn (M_m) to sun vertical depression angle (D_o),

$$D_o = \sin^{-1} [\sin \varphi \sin \delta + \cos \varphi \cos \delta \cos H]$$

$$H = 15[Noon - M_m] \quad \text{and} \quad Noon = 12 - \frac{ET}{60} - \frac{\Delta\lambda}{15}$$

$$E.T.(\text{min}) = 9.87 \sin(2B) - 7.53 \cos B - 1.5 \sin B$$

$$B = \left[\frac{360}{365} (n - 81) \right]$$

$$n = \left[\frac{275 m}{9} \right] - k \left[\frac{m + 9}{12} \right] + d - 30$$

$$\delta = 23.45 \sin \left[\frac{360}{365} (284 + n) \right]$$

To calculate the dawn time (M_c) throughout the year in any country as a function of the sun vertical depression (dawn time for any day of the year)

$$M_c = Noon - \frac{H}{15} \quad \text{and} \quad H = \cos^{-1} \left[\frac{\cos Z - \sin \varphi \sin \delta}{\cos \varphi \cos \delta} \right]$$

Appendix B

B.1. The concept of duration of twilight Δt

We should highlight some of the concepts and terms. The relation between the sun vertical depression (D_o) and the duration of twilight Δt is simply given by the equation,

$$\Delta t_{(\varphi, \delta)} = \frac{H - H'}{15}$$

The arc degree equals 4 min only at the equator in the equinox ($\delta = 0^\circ$ and $\varphi = 0^\circ$), but if the latitude increases, the arc degree increases steadily and the arc degree exceeds also steadily.

At sunrise or sunset, $Z = 90^\circ$. Then the hour angle at the center of the sun disk H' is as follows:

$$H' = \cos^{-1} [-\tan \varphi \tan \delta]$$

But at any D_o , the hour angle of the sun is:

$$H = \cos^{-1} \left[\frac{\cos Z - \sin \varphi \sin \delta}{\cos \varphi \cos \delta} \right]$$

And the variation of D_o with time, $\frac{dD_o}{dt}$ (Smart, 1977 and Patat et al., 2006) is:

$$\frac{dD_o}{dt} = \frac{dH}{dt} \cos D_o \sin H \cos \delta \cos \varphi$$

Then the duration of twilight is simply as

If we assume $Z = 105^\circ$ ($D_o = 15^\circ$) for the dawn,

$\Delta t_{(0,0)} = 1$ h, then the one degree ($1 \times 60/15$) $\equiv 4$ min at the equator and in the equinox, then $\frac{dH}{dt} = 0.25$ degree min^{-1} .

$\Delta t_{(48.5, -23.45)} = 1.065$ h, then the one degree ($1.065 \times 60/15$) $\equiv 4.26$ min at ($\varphi = 48.5^\circ$ and $\delta = -23.45^\circ$), then $\frac{dH}{dt} = 0.235$ degree min^{-1} .

$\Delta t_{(48.5, +23.45)} = 1.27667$ h, then the one degree ($1.27667 \times 60/15$) $\equiv 5.107$ min at ($\varphi = 48.5^\circ$ and $\delta = +23.45^\circ$), then $\frac{dH}{dt} = 0.196$ degree min^{-1} .

So, the time of the twilight rises with latitude φ and sun declination δ .

On the opposite, one degree in case the sun vertical depression angle (D_o) does not equal 4 min except at the equator and in the equinoxes. So the sustainability of twilight reflects the real-time between dawn and sunrise.

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