Full length article

Impact of Nasser Lake on gravity reduction and geoidal heights for Egypt

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A B S T R A C T

In the course of the IAG African Geoid Project, it is needed to study the impact of the lakes on the gravity reduction and geoidal heights. The aim of this paper is to study the impact of the water in Nasser Lake on gravity reduction and geoidal heights for Egypt. The determination of the gravimetric geoid is based on the well-known remove-restore technique. The problem of the lakes occurs because the popular programs widely used in practice (e.g., TC-program (Forsberg, 1984)) assume that all positive elevations are filled with rock topography, and all negative elevations are filled with ocean water. This is, however, not true for the case of Nasser Lake, which lies completely above sea level, at about 180 m elevation, with a water depth of about 20 m. The paper presents an approach on estimating the impact of Nasser Lake on gravity reduction and geoidal heights using TC-program with some tricky cases. The results show that the impact of Nasser Lake on both gravity anomalies and geoid undulation is limited to the area of the lake. The impact of Nasser Lake on the gravity anomalies is in the order of sub mgal, while the impact of Nasser lake on the geoid undulation is significant and reaches few centimeters.

1. Introduction

In order to apply Laplace equation, there should be no masses outside the geoid (Heiskanen and Moritz, 1967). This implies removing, mathematically, the topographic masses and their compensating masses. This process is known in physical geodesy as gravity reduction. The remove-restore technique (Forsberg, 1984) is widely used in practice. Abd-Elmotaal and Kühtreiber (2003) have proposed a more accurate remove-restore technique by introducing the so-called window technique. In both techniques, the effect of the topographic masses are removed using TC-program (written originally by Forsberg (1984)) or its modified version (written by Abd-Elmotaal and Kühtreiber (2003)). As TC-program uses unclassified Digital Height Models (DHMs), it assumes that all positive elevations are filled with rock topography, and all negative elevations are filled with ocean water.

In the case of Nasser Lake, the situation is quite different. The whole lake is located above the sea level. Thus it has positive elevations. TC-program thus treats the water masses inside the Nasser Lake as rock topography instead of water masses. Accordingly, the main aim of the current investigation is to determine the impact of the Nasser Lake on both the gravity reduction and the geoid undulation.

The study area is defined and an overview of the Nasser Lake is given. The available Digital Height Models for the current investigation are described. The Stokes' technique of the geoid determination, within the remove-restore scheme, is outlined. The used images for determining the lake and the necessary image analysis has been given. The approach of determining the impact of the Nasser Lake on the gravity reduction and the geoid undulation using TC-program is explained and then implemented.

It should be noted that the area of Lake Nasser attracts a number of interesting studies. Some studies were interested in computing a local geoid for the Nasser Lake area (see, e.g., Tscherning et al., 2001). Other studies were tackling the important topic of the crustal deformation in the area of Lake Nasser (cf., e.g., Zahran and Abdel-Monem, 2006).

2. Study area – Nasser Lake

Nasser Lake is one of the largest artificial freshwater reservoirs in the world. It is considered as the largest artificial reservoir in Africa by volume (excluding continental lakes, e.g., Tanganyika). The Lake is located in southern Egypt and northern Sudan. Strictly, "Lake Nasser" refers only to the much larger portion of the lake that is in the Egyptian territory (83% of the total area), (cf. Fig. 1). The Sudanese prefer to call their smaller portion of the lake "Lake Nubia". This study concerns the whole area of the lake. The lake was created after the construction of the High Dam across the River Nile waters during the 1960s. The lake lies between latitudes $21.75°N \leq \phi \leq 24°N$, extends over 500 km long upstream of the High Dam, and has a width of 35 km at its widest point (near the Tropic of Cancer). At 182 m water level (above mean sea level), the lake has an average width of about 12 km and covers a

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Fig. 1. Lake Nasser location.

surface area of 5237 km^2 with a storage capacity of some 150–165 km^3 of water (Zwieten et al., 2011).

Nasser Lake is valuable for Egypt as it represents a large reservoir for the country's freshwater resources. The location of the lake is characterized by an extremely arid climate, with a wide diurnal variations and almost no precipitation. The volume of the water in the lake fluctuates greatly annually and seasonally depending upon the net annual volume of water it receives and the operation of the High Dam. Fig. 2 shows the lake level fluctuation as inferred from satellite altimetry (LEGOS/GOHS) from 1992 to 2011. The highest recorded water level during this period was 181.2 m (above mean sea level) in November 1999, while the lowest level recorded so far was 167.2 m in July 1993. In this study, satellite images, namely Landsat-7 ETM + images, are used to determine the boarders of the lake as described in Section 7.

3. Digital height models DHMs

In the context of the remove-restore technique (Forsberg, 1984), a set of fine and coarse DHMs is needed for the terrain reduction computation. The $3'' \times 3''$ fine Digital Height Model EGH13S03 and the $30'' \times 30''$ coarse Digital Height Model EGH13S30 (Abd-Elmotaal et al., 2013) are available for this investigation. These DHMs cover the full window of $(19°N \leq \phi \leq 35°N; 22°E \leq \lambda \leq 40°E)$. Fig. 3 illustrates the $3'' \times 3''$ fine EGH13S03 for the Lake Nasser area

$(20°N \leq \phi \leq 26°N; 30°E \leq \lambda \leq 36°E)$. The heights within the Nasser Lake area range between -1776 m and 1961 m with an average of about 289.1 m and a standard deviation of 293.8 m.

4. Remove-restore technique

The remove-restore technique consists of two steps. The first step is performed to remove the effect of the topographic-isostatic masses and the effect of the global geopotential model from the source gravitational data. In the second step, the effect of the topographic-isostatic masses and the effect of the global geopotential model to the resulting geoidal heights are restored. For example, in the case of gravity data, the reduced gravity anomalies in the framework of the remove-restore technique are computed by (see e.g., Rapp and Rummel (1975), Forsberg (1993), and Sansó (1997))

$$\Delta g_{red} = \Delta g_F - \Delta g_{GM} - \Delta g_{TI}, \tag{1}$$

where Δg_F stands for the free-air anomalies, Δg_{TI} is the effect of topography and its compensation on the gravity anomalies, and Δg_{GM} is the effect of the reference field on the gravity anomalies. Thus, the final computed geoid N within the remove-restore technique can be expressed by

$$N = N_{\Delta g} + N_{GM} + N_{TI}, \tag{2}$$

where N_{GM} stands for the contribution of the reference field, $N_{\Delta g}$ stands for the contribution of the reduced gravity anomalies, and N_{TI} is the contribution of the topographic-isostatic masses (the so-called indirect effect).

The above *traditional* remove-restore techniques faces a theoretical problem of the double consideration of some of the topographic-isostatic masses around the computational point. A possible solution of this problem has been made by introducing the *window* remove-restor technique (Abd-Elmotaal and Kühtreiber, 1999, 2003).

The remove step for the window remove-restor technique is given by

$$\Delta g_{win-red} = \Delta g_F - \Delta g_{GM} - \Delta g_{TI\,win} + \Delta g_{wincof}, \tag{3}$$

where $\Delta g_{win-red}$ stands for the window-reduced anomalies, $\Delta g_{TI\,win}$ refers to the effect of the topographic-isostatic masses for a fixed full data window and Δg_{wincof} is the contribution of the harmonic coefficients of the topographic-isostatic masses of the data window. The harmonic coefficients of the topographic-isostatic masses can be computed using the expressions given in Abd-Elmotaal and Kühtreiber (2003), or more precisely using the expression given in Abd-Elmotaal and Kühtreiber (2015).

The restore step for the window remove-restor technique is given by

$$N = N_{\Delta g\,win} + N_{GM} + N_{TI\,win} - N_{wincof}, \tag{4}$$

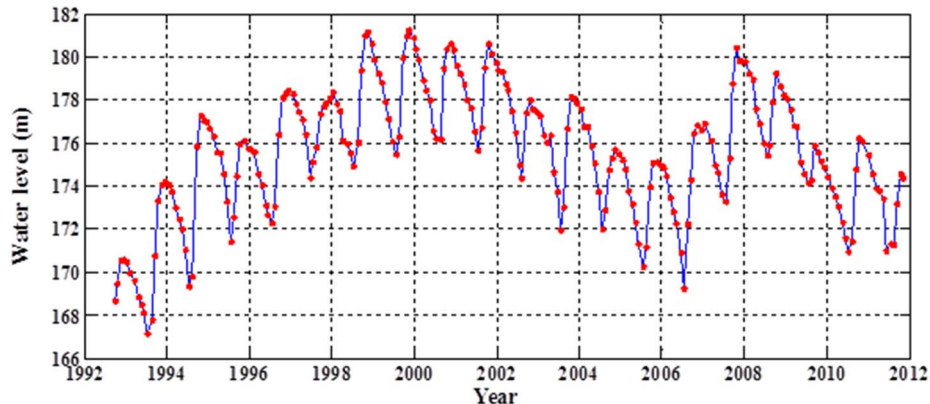where $N_{\Delta g\,win}$ stands for the contribution of the window-reduced gravity

Fig. 2. Lake level variation of Lake Nasser from satellite altimetry (LEGOS/GOHS) (reference point: $\phi = 22.60°, \lambda = 32.10°$).
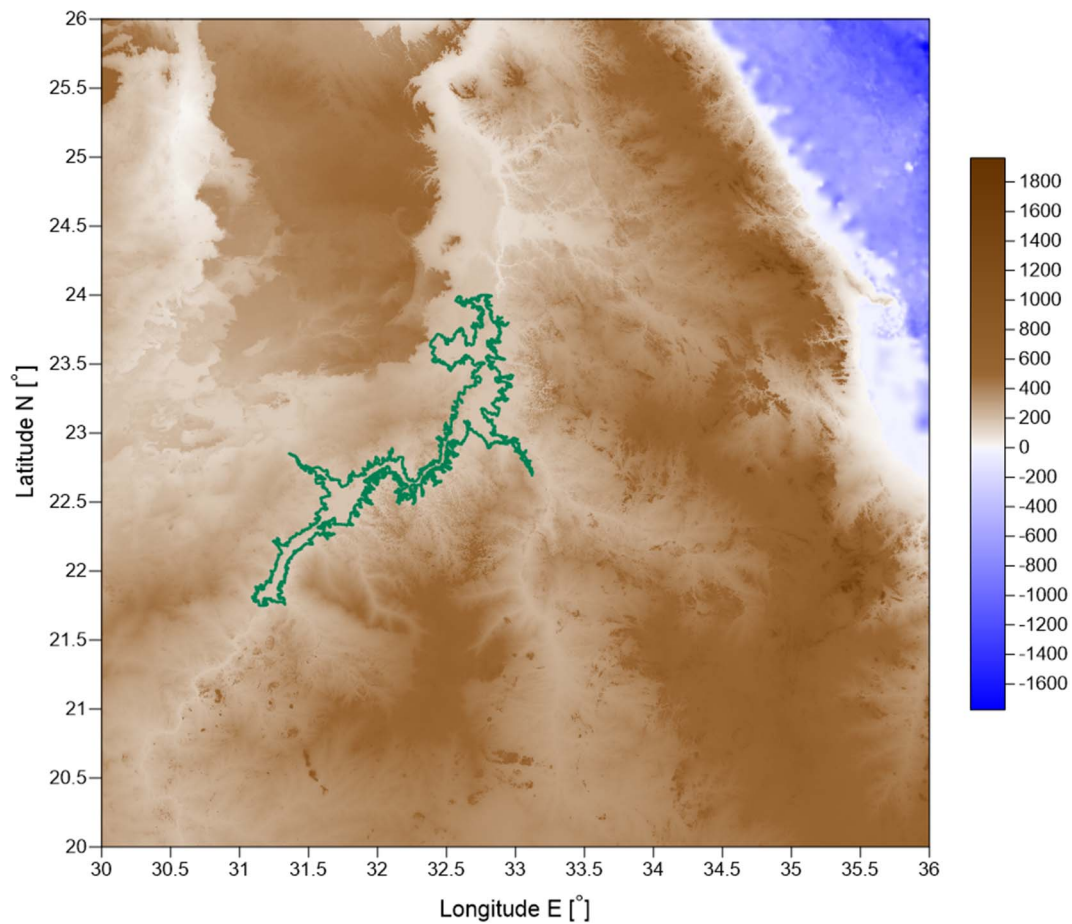
Fig. 3. The $3'' \times 3''$ fine Digital Height Model EGH13S03 (after Abd-Elmotaal et al. (2013)) for the Nasser Lake area. Units in [m].
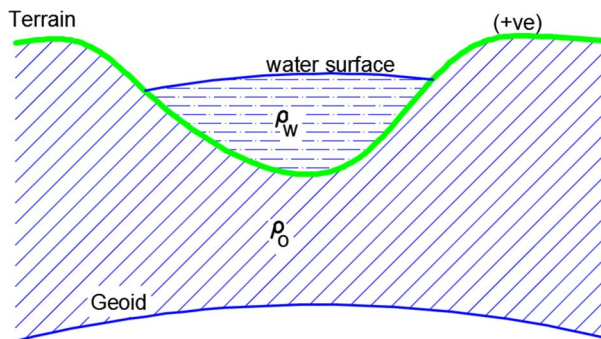

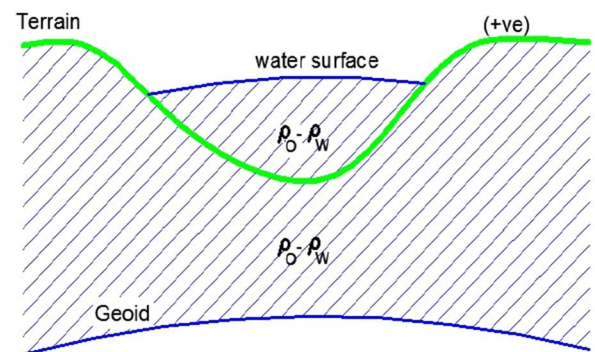**Fig. 4.** The original case: the lake is completely located above the mean sea level.
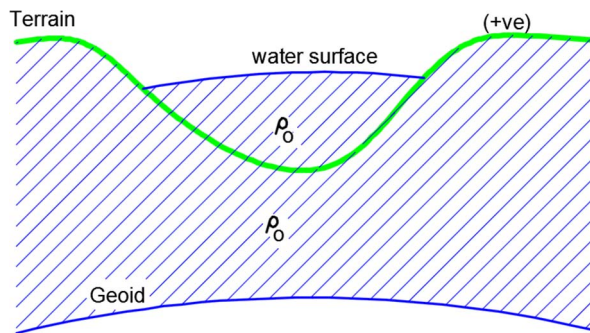

Fig. 6. Case (1) of the suggested approach.

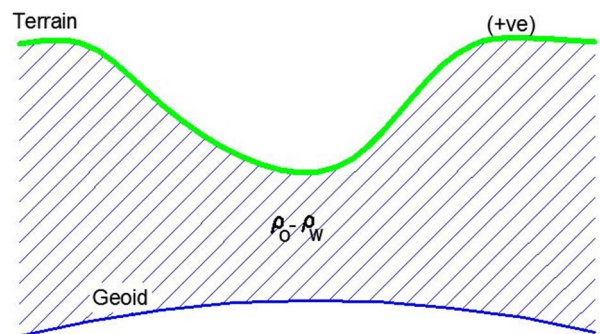**Fig. 5.** The "wrong" case assumed by TC-program with unclassified DHMs.

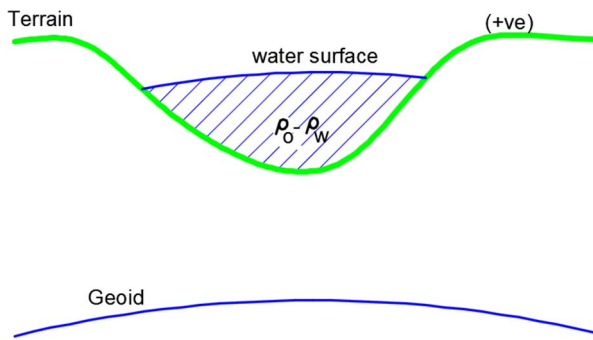**Fig. 7.** Case (2) of the suggested approach.

Fig. 8. The difference between case (1) and case (2) equals the impact of Nasser Lake.

Table 1
Acquiring data of Landsat images.

Path/row	P174/R044	P175/R044	P175/R045
Acquisition date	Aug 24, 2011	Aug 31, 2011	Aug 31, 2011
Scene time	08:06:52	08:13:02	08:13:26
SLC	Off	Off	Off
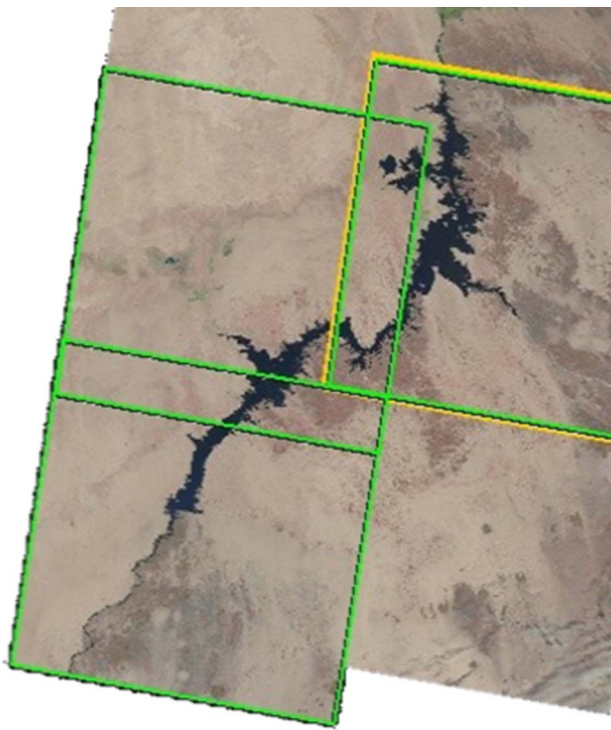
Fig. 9. Locations (in path/row) of the used Landsat images in WRS-2 of Landsat-7.

Fig. 10. Classification results for the considered Landsat images showing the water class representing Lake Nasser.

Fig. 11. Water class vector representing Lake Nasser as extracted from ArcGIS system.

anomalies, and $N_{TI\,win}$ is the contribution of the topographic-isostatic masses for the fixed data window and N_{wincof} is the contribution of the harmonic coefficients of the topographic-isostatic masses of the data window.

The computation within the window remove-restore technique is rather time consuming. This depends essentially on the resolution of the used DHMs. Accordingly, one needs to define the optimal DHM resolutions for both the terrain reduction (using TC-program) and the computation of the harmonic coefficients of the topographic-isostatic masses of the data window. The reader is then kindly invited to refer to Abd-Elmotaal (2011) and Abd-Elmotaal and Kühtreiber (2014).
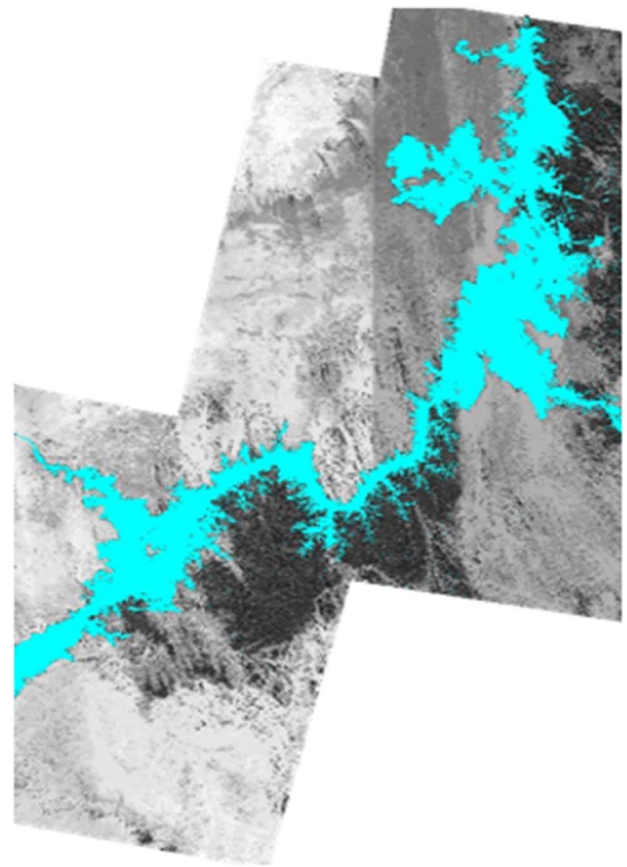
5. Stokes' technique

The Stokes' equation expresses the geoid undulation in terms of a convolution of the gravity anomalies with the Stokes' function. Using

Fig. 12. The impact of the Nasser Lake on the gravity anomaly. Units in [mgal].

Stokes' equation, the contribution of the reduced gravity anomalies $N_{\Delta g}$ is given by (Heiskanen and Moritz, 1967)

$$N_{\Delta g} = \frac{R}{4\pi\gamma} \iint_{\sigma} \Delta g_{red} S(\psi) \, d\sigma, \tag{5}$$
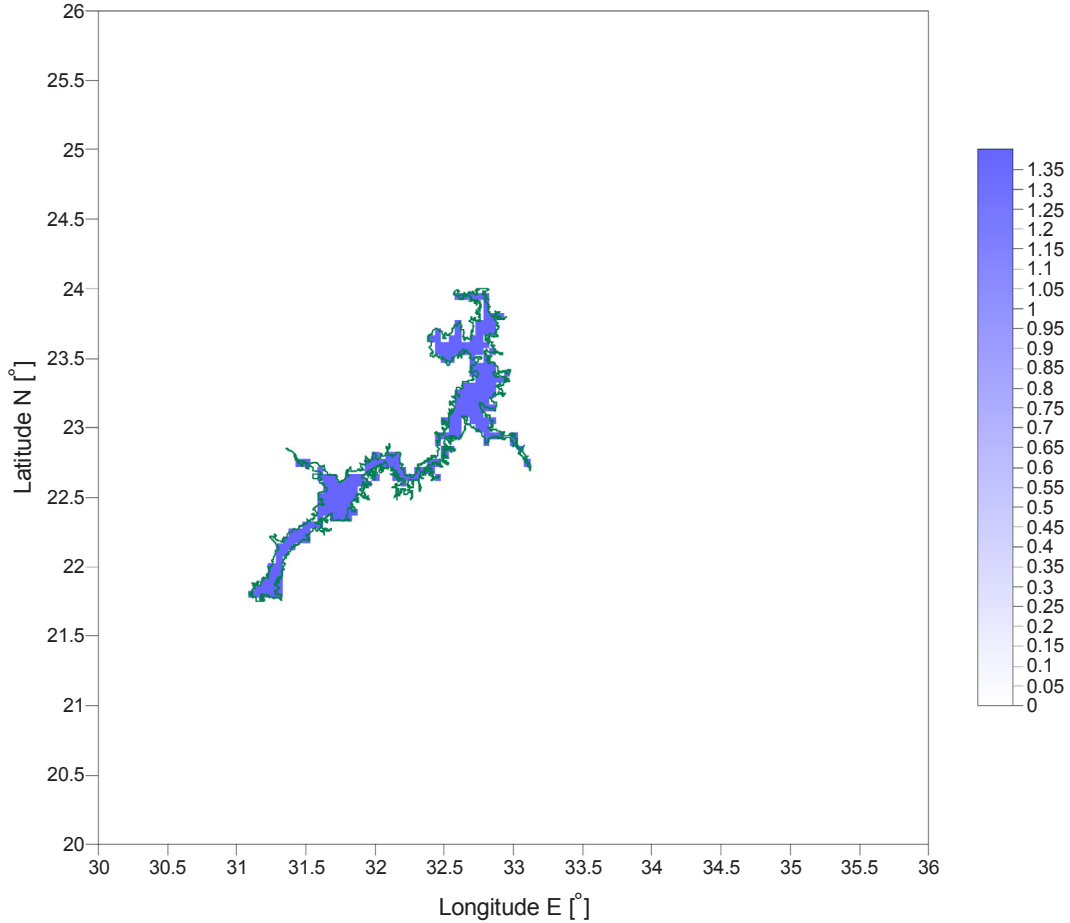
where γ is the normal gravity, R is the mean Earth's radius and $S(\psi)$ stands for the Stokes un-modified kernel given by (Strang van Hees, 1990)

$$S(\psi) = \frac{1}{s} - 4 - 6s + 10s^2 - (3 - 6s^2)\ln(s + s^2). \tag{6}$$

with

$$s = \sin\frac{\psi}{2}, \tag{7}$$

and ψ is the spherical distance between the computational point P and the running point Q. The evaluation of the integral in Eq. (5) is performed by numerical integration. Hence, the integral is replaced by a summation over two parameters covering theoretically the whole surface of the Earth. Thus, Eq. (5) becomes

$$N(\phi_p, \lambda_p) = \frac{R\Delta\phi\Delta\lambda}{4\pi G} \sum_{Q} (\Delta g_Q \cos\phi_Q) S(\psi_{PQ}). \tag{8}$$

More details can be found, e.g., in Strang van Hees (1990) and Abd-Elmotaal (2014).

6. Methodology

The elevation of the water in Nasser Lake is about 182 m above the sea level. As shown in Fig. 4, the masses above the geoid consist of two different material; rock topography with an average density of 2.67 g/cm³, and water with density of about 1.03 g/cm³.

The TC program (Forsberg, 1984), after the significant modifications made by Abd-Elmotaal and Kühtreiber (2003), has been used to compute the effect of topographic masses on both the gravity anomalies and the geoid undulation. As mentioned above, TC-program uses unclassified DHMs, and thus treats the masses above the geoid as rock topography with an average density of 2.67 g/cm³ (cf. Fig. 5).

The following two cases have been considered to compute the correct impact of the Nasser Lake on the gravity anomalies and geoid undulation:

1. Case (1): Consider the masses above the geoid with a constant density of $\rho_o - \rho_w$ (cf. Fig. 6).
2. Case (2): Remove the lake part from the DHM and then consider the masses above the geoid of density $\rho_o - \rho_w$ (cf. Fig. 7).

Accordingly, by subtracting case (2) from case (1), one gets the impact of the Nasser Lake on the gravity reduction and geoid undulation (cf. Fig. 8).

7. Image analysis

In this study, three scenes of Landsat-7 images covering the entire lake are used. The images are available on USGS Global Visualization Viewer (GloVis): http://glovis.usgs.gov/. Table 1 shows the details of the used images. Lake Nasser is covered by three scenes located in two paths (namely; P174 and P175) as shown in Fig. 9.

In this work, the used images are SLC-off images (cf. Table 1). In order to fill the gaps in SLC-off images, it is possible to combine two
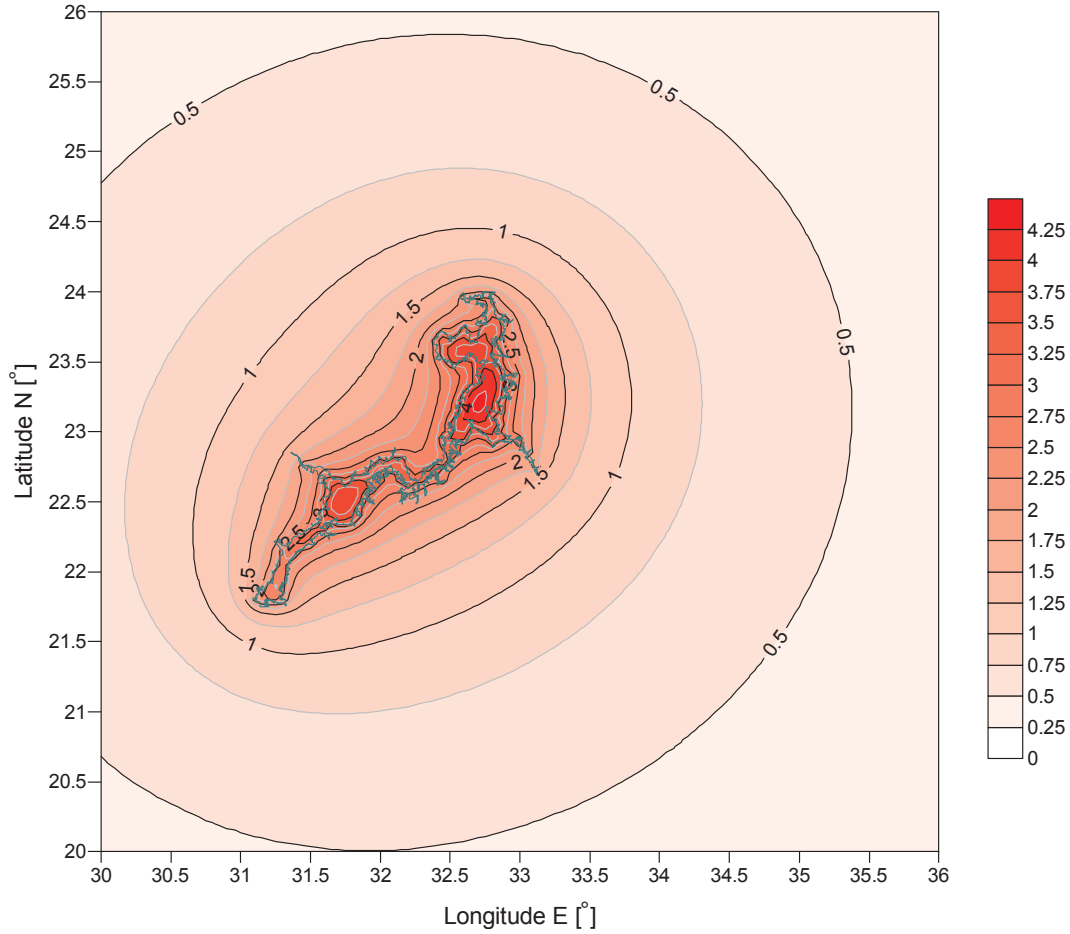
Fig. 13. The impact of the lake on the geoid undulation. Units in [cm].

SLC-off scenes (primary scene and fill scene) to create a somewhat completely filled final image (Chen et al., 2011). The choice of the primary and fill scenes should be based on specific criteria, as follows:

- Both primary and fill scenes should be cloud free as far as possible,
- The date of the fill scene should be as close as possible to the date of the primary scene. Here, the fill scene is chosen to be directly following the primary scene during the same month. The time difference between the two successive scenes is the repeat-cycle period of Landsat-7 (16 days),
- The area of interest should contain low temporal variability with respect to land cover and surface features, and
- There should be minimal differences in transient conditions between the primary and fill scenes.

Filling gaps is done for this work using ERDAS Imagine TM model-maker tool. The three scenes are then mosaicked and corrected geometrically using ERDAS Imagine. The output image is then classified with an unsupervised classification tool, namely Iterative Self Organizing Data Analysis (ISODATA), in order to obtain the water body class of the lake. In the absence of field measurements, unsupervised classification is an effective technique for water texture recognition, especially in such arid regions. The ISODATA utility repeats the clustering of the image until the percentage of convergence threshold is reached. The convergence threshold is the maximum percentage of pixels whose cluster assignments can be unchanged between two successive iterations. Here, the convergence threshold is set as 0.97, which implies that as soon as 97% or more of the pixels stay in the same cluster between two successive iterations, the clustering utility stops

processing. Fig. 10 shows the classified images with the water class (representing the lake area) shown in cyan. The water class is then extracted as a vector format in using ArcGIS system. Fig. 11 shows the water class vector representing Lake Nasser as extracted from ArcGIS system.

8. Results and discussions

For estimating the impact of the Nasser Lake on the gravity anomalies and the geoidal heights in Egypt, the effect of the topographic masses of the fixed full data window $(19°N \leq \phi \leq 35°N; 22°E \leq \lambda \leq 40°E)$ have been computed using the TC-program employing the Digital Height Models discussed above on a $3' \times 3'$ grid covering the full data window. The fine EGH13S03 DHM has been used till a radius R_1 of 6 km around the computational point, hence the coarse EGH13S30 DHM has been used for the rest of the full data window. The following parameters have been used because they proved practically to give the best results for Egypt (Abd-Elmotaal, 2009)

$T_o = 30$ km,
$\rho_o = 2.67$ g/cm^3,
$\Delta\rho = 0.40$ g/cm^3, $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (9)

where T_o is the normal crustal thickness, ρ_o is the density of the topography and $\Delta\rho$ is the density contrast between the crust and the mantle.

The density $\rho_o - \rho_w$ of the topographic masses for case (1) has been taken to be 1.64 g/cm^3. Then, the Digital Height Models have been modified by subtracting the depths of the water in Nasser Lake region from the heights of the original Digital Height Models (representing the surface of the lake). Thus case (2) took place and the effect of the

modified Digital Height Models on the gravity anomalies and the geoid heights have been computed at the same grid points as of case (1) having the same value for the density $\rho_o - \rho_w$. The differences between the results in the two cases have been computed and will be shown and discussed in the following sections.

8.1. Effect on gravity anomalies

Fig. 12 shows the impact of the Nasser Lake waters on the gravity anomaly. Fig. 12 shows significantly small effects on the gravity anomalies near and inside the lake and zero effect elsewhere. The effect of the Nasser Lake for the study area ($20°N \leq \phi \leq 26°N$ and $30°E \leq \lambda \leq 36°E$) ranges between zero and 1.4 mgal with an average of 0.02 mgal and a standard deviation of 0.17 mgal.

8.2. Effect on geoid undulation

The effect of the gravity anomalies on geoidal heights has been computed using Stokes' formula (8) on a $3' \times 3'$ grid covering the full window ($19°N \leq \phi \leq 35°N; 22°E \leq \lambda \leq 40°E$). Fig. 13 shows the impact of the Nasser Lake on the geoid undulation. It shows that the impact of the Nasser Lake on the geoid undulation is about 4 cm near and inside the lake. The effect of Nasser Lake for the study area ($20°N \leq \phi \leq 26°N$ and $30°E \leq \lambda \leq 36°E$) ranges between 0.3 cm and 4.3 cm with an average of 0.8 cm and a standard deviation of 0.55 cm. The indirect effect of the lake is very small and can be neglected.

9. Conclusions

The current investigation studied a very important point in the process of the geoid determination. It is the impact of the lakes. An approach using the widely used in practice TC-program to determine the impact of the Nasser Lake on the gravity anomalies and geoid undulation has been developed within the context of the current paper. In addition, Landat-7 ETM+ images are processed to precisely determine the boarders of the lake. The developed approach has successfully been implemented to compute the effect of Nasser Lake on gravity reduction and geoid determination. The results reveal that the impact of Nasser Lake on the gravity anomalies and the geoid undulations is limited only to the area of the Nasser Lake. The impact of the Nasser Lake on the gravity anomalies within the area of the lake is in the order of only sub mgal. However, the impact of the Nasser Lake on the geoid undulation is significant and reaches few centimeters, which is then needed to be considered for the cm-geoid.

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